姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103 / GEAI 1215: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 7 & 3 \\ 2 & -1 & 11 & 6 \\ -5 & 1 & -23 & -15 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_1 = \begin{bmatrix} -4 \\ 3 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 0.

NullBasis 1

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

0

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Quiz 1 MATH 103 / GEAI 1215: Linear Algebra I

Consider the equation $\mathbf{Ax} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 5 & -5 & -5 \\ 4 & 20 & -20 & -19 \\ 11 & 55 & -55 & -53 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 5 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_3.

By setting $x_2 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_1 = \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 6.

NullBasis 2

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

6

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103 / GEAI 1215: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -5 & -5 & -6 \\ -5 & 26 & 26 & 31 \\ 19 & -98 & -98 & -117 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 0.

NullBasis 3

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

0

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103 / GEAI 1215: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & 4 \\ 5 & -4 & -4 & 15 \\ 18 & -15 & -12 & 57 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -4 & -1 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_1 = \begin{bmatrix} 4 \\ 4 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 9.

NullBasis 4

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

9

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Quiz 1 MATH 103 / GEAI 1215: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 3 & 18 \\ 5 & 15 & 16 & 95 \\ -5 & -15 & -15 & -90 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_4.

By setting $x_4 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} -3 \\ 0 \\ -5 \\ 1 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 3.

NullBasis 5

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

3

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103 / GEAI 1215: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 0 & -3 \\ -5 & -20 & 0 & 16 \\ 2 & 8 & 0 & -6 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_3.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 1.

NullBasis 6

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

1

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103 / GEAI 1215: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 & 2 \\ -5 & 6 & -10 & -10 \\ 25 & -30 & 50 & 50 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -4 & 2 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_1 = \begin{bmatrix} 4 \\ 5 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 0.

NullBasis 7

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

0

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Quiz 1 MATH 103 / GEAI 1215: Linear Algebra I

Consider the equation $\mathbf{Ax} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & -1 & 11 \\ 5 & -14 & -3 & 50 \\ 7 & -19 & -3 & 67 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 5 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_1 = \begin{bmatrix} -5 \\ -2 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 4.

NullBasis 8

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

4

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103 / GEAI 1215: Linear Algebra I

Consider the equation $\mathbf{Ax} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & -5 & -24 \\ 0 & 0 & 1 & 4 \\ 4 & -4 & -25 & -116 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & -4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_4.

By setting $x_4 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} 4 \\ 0 \\ -4 \\ 1 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 1.

NullBasis 9

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

1

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 3 & -8 \\ 4 & -4 & 13 & -35 \\ -14 & 14 & -46 & 124 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_4.

By setting $x_4 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} -1 \\ 0 \\ 3 \\ 1 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 3.

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

3