Sample Questions 10

1. Let $\mathbf{A}$ be an $n \times n$ matrix with $\det(\mathbf{A}) = 2$. Find $\det(\operatorname{adj}(\mathbf{A}))$.

2. Let
$$\mathbf{A} = \begin{bmatrix} 0 & a & 0 & 0 & e \\ a & 0 & b & 0 & 0 \\ 0 & b & 0 & c & 0 \\ 0 & 0 & c & 0 & d \\ e & 0 & 0 & d & 0 \end{bmatrix}.$$
Show that $\mathbf{A}$ is invertible when each of a, b, c, d, e is a nonzero real number.

3. Let
$$\mathbf{A}(t) = \begin{bmatrix} t & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$
and $\det(\mathbf{A}(t)) = at + b$. Find a.

4. Let
$$\mathbf{A}(t) = \begin{bmatrix} 1-t & 1 & 1 \\ 1 & 1-t & 1 \\ 1 & 1 & 1-t \end{bmatrix}.$$
Suppose $\mathbf{A}(t)\mathbf{x} = \mathbf{0}$ has nonzero solution. Find all possible t and the corresponding nullspaces.

5. Let
$$\mathbf{A}(t) = \begin{bmatrix} 2-t & -1 & -1 \\ -1 & 1-t & 0 \\ -1 & 0 & 1-t \end{bmatrix}.$$
Suppose $\mathbf{A}(t)\mathbf{x} = \mathbf{0}$ has nonzero solution. Find all possible t and the corresponding nullspaces.

Let
$$\mathbf{A}(t) = \begin{bmatrix} -t & 1 & 0 & 0 & 0 \\ 1 & -t & 1 & 0 & 0 \\ 0 & 1 & -t & 1 & 0 \\ 0 & 0 & 1 & -t & 1 \\ 0 & 0 & 0 & 1 & -t \end{bmatrix}$$
and $\det(\mathbf{A}(t)) = \sum_{k=0}^{5} a_k t^k$.

6. Find a_5 and a_0.

7. Find a_4 and a_1.