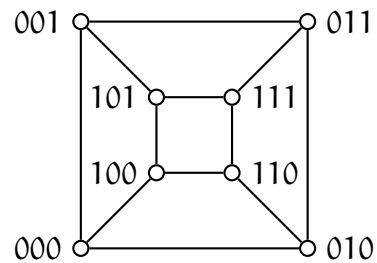
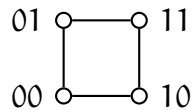


Math589 Homework 3

Note: To submit the k-th homework, simply put your files in the folder HWk on CoCalc, and it will be collected on the due day.

1. The *Hamming distance* between two 0, 1-strings is the number of different digits. For example, the Hamming distance between 010101 and 111000 is 3. The *hypercube* H_d of dimension d has vertices as all 0, 1-strings of length d , and two vertices are adjacent if the Hamming distance of the strings is 1. The graphs below illustrate H_2 and H_3 . Show that H_d is a bipartite graph for all d .



2. Let K_n be the complete graph on n vertices. Find the number of spanning trees on K_n by the following way: Let L_n be the Laplacian matrix of K_n . Recall that $L_n(1, 1)$ is the matrix obtained from L_n by removing the first row and the first column. Then the number of spanning tree equals $\det L_n(1, 1)$.
[Hint: Think about the the eigenvalues of J , the all-ones matrix.]