Math589 Homework 7

Note: To submit the k-th homework, simply put your files in the folder `HWk` on CoCalc, and it will be collected on the due day.

1. Let $[n] = \{1, \ldots, n\}$. For any subset $\alpha \subseteq [n]$, the characteristic vector ϕ_α of α is a vector in $\mathbb{R}^n$ whose i-th entry is 1 if $i \in \alpha$ and 0 otherwise. Show that $\{\phi_\emptyset, \phi_{[1]}, \ldots, \phi_{[n]}\}$ is affinely independent.

2. Let the characteristic vectors be defined as in the previous question with $n = 3$. Let π be a permutation on $\{1, 2, 3\}$. Define a simplex

$$S_\pi = \mathrm{conv}(\{\phi_\emptyset, \phi_{\{\pi(1)\}}, \phi_{\{\pi(1), \pi(2)\}}, \phi_{\{\pi(1), \pi(2), \pi(3)\}}\}).$$

We showed that S_π is a simplex for $\pi = \mathrm{id}_{[3]}$. Indeed, S_π is a simplex for any permutation π. (You do not have to show this.) Show that the cubic enclosed by

$$0 \leqslant x_1, x_2, x_3 \leqslant 1$$

is the union of S_π for all permutation π.