

Math589 Homework 9

Note: To submit the k-th homework, simply put your files in the folder HWk on CoCalc, and it will be collected on the due day.

1. Let F be a subset of $\{e_1, -e_1, \dots, e_d, -e_d\}$. Show that $\text{conv}(F)$ is a proper face of the crosspolytope if and only if there is no i such that both e_i and $-e_i$ are in F .

2. Let

$$v_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \text{ and } v_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

Let Δ be the simplicial complex composed of the simplices

$$\text{conv}(\{v_0, v_1, v_2\}), \text{conv}(\{v_0, v_2, v_3\}), \text{conv}(\{v_0, v_3, v_4\}), \text{conv}(\{v_0, v_4, v_1\}),$$

and their faces. Define $f: V(\Delta) \rightarrow \mathbb{R}^2$ by

$$f(v_0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, f(v_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, f(v_2) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, f(v_3) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \text{ and } f(v_4) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Find an exact formula for the affine extension $\|f\|$ of f . That is, what is

$$\|f\| \left(\begin{bmatrix} x \\ y \end{bmatrix} \right)?$$