

Math589 Midterm2

6 questions, 24 total points

Note: Use other papers to answer the problems. Remember to write down your **name** and your **student ID #**.

1. [4pt] Show that the Kneser graph $K_{7,3}$ is not 2-colorable.

2. [4pt] Let (X, \mathcal{O}) be a topological space with

$$X = \{1, 2, 3, 4, 5\} \text{ and } \mathcal{O} = \{\emptyset, X, \{1\}, \{2\}, \{1, 2\}\}.$$

Let $Y = \{1\}$.

- Describe all closed sets on X .
- Find the closure $\text{cl}(Y)$.
- Find the boundary ∂Y .
- Find the interior $\text{int}(Y)$ of Y .

3. [4pt] Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ 4 \\ 10 \end{bmatrix}, \text{ and } \mathbf{v}_4 = \begin{bmatrix} 2 \\ 5 \\ 17 \end{bmatrix}.$$

Show that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is affinely independent.

4. [4pt] Let $[3] = \{1, 2, 3\}$. For any subset $\alpha \subseteq [3]$, the characteristic vector ϕ_α of α is a vector in \mathbb{R}^3 whose i -th entry is 1 if $i \in \alpha$ and 0 otherwise. Let π be a permutation on $\{1, 2, 3\}$. Define a simplex

$$S_\pi = \text{conv}(\{\phi_\emptyset, \phi_{\{\pi(1)\}}, \phi_{\{\pi(1), \pi(2)\}}, \phi_{\{\pi(1), \pi(2), \pi(3)\}}\}).$$

Then the cube enclosed by

$$0 \leq x_1, x_2, x_3 \leq 1$$

is the union of S_π for all permutation π . (You do not have to show this.) Let $\mathbf{v} = (0.2, 0.7, 0.3)^\top \in \mathbb{R}^3$ be a point in the cube. Which simplex S_π does \mathbf{v} belong to?

[More questions on the back]

5. [4pt] What is a simplex? What is a simplicial complex?

6. [4pt] Let C_4 be the cycle on 4 vertices. Let L be the Laplacian matrix of C_4 . Find the eigenvalues and an eigenbasis of L .