姓名 Name　:　_____

學號 Student ID #　:　_____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **5 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

– Enter your **Name** and **Student ID #** before you start.

– Using the calculator is not allowed (and not necessary) for this exam.

– Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

– Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

– 可用中文或英文作答

1. [5pt] Recall that $H_k^m = \binom{m+k-1}{k}$ counts the number of integer solutions of

$$x_1 + \cdots + x_m = k, \quad x_i \geq 0$$

for all $i = 1, \ldots, m$. **Use double counting** to prove that

$$H_k^{m+1} = H_k^m + H_{k-1}^m + \cdots + H_0^m.$$

2. [5pt] **Use mathematical induction** to prove that

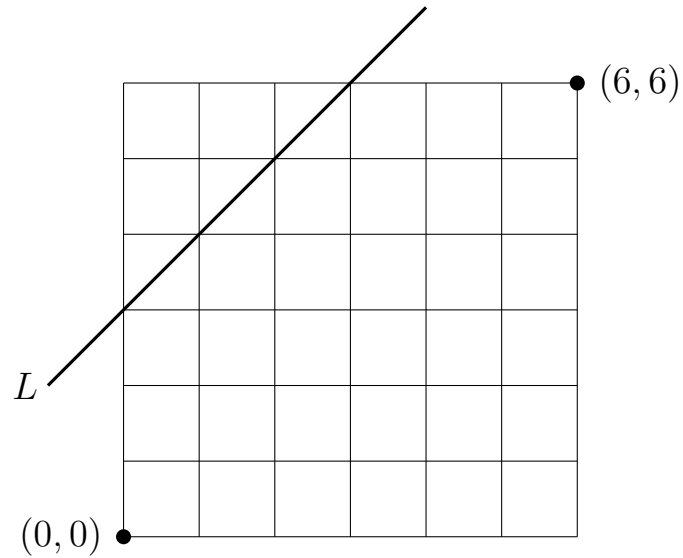$$m^3 + 2m \text{ is divisible by 3 for all integer } m \geq 1.$$

3. [5pt] Prove that any set $S \subseteq \{1, \ldots, 120\}$ with $|S| = 61$ contains two numbers a and b such that a is divisible by b.

4. [5pt] Let $m = 2021$ and $n = 110$. Find integers a and b such that

$$am + bn = 1.$$

5. [extra 2pt] Consider two possible moves $\rightarrow$: $(1,0)$ and $\uparrow$: $(0,1)$. Count the number of ways to go from $(0,0)$ to $(6,6)$ such that

- each step is either $\rightarrow$ or $\uparrow$, and
- it **touches** the line $L : y = x + 3$. (有碰到就算)

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	