

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

離散數學 (一)

MATH 203: Discrete Mathematics I

第一次期中考

October 13, 2020

Midterm 1

姓名 Name : solation

學號 Student ID # : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, 5 pages of questions, score page at the end
To be answered:	on the test paper
Duration:	110 minutes
Total points:	20 points + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. [5pt] Recall that $H_k^m = \binom{m+k-1}{k}$ counts the number of integer solutions of

$$x_1 + \cdots + x_m = k, \quad x_i \geq 0$$

for all $i = 1, \dots, m$. Use **double counting** to prove that

$$H_k^{m+1} = H_k^m + H_{k-1}^m + \cdots + H_0^m.$$

See Ver. A.

2. [5pt] Use **mathematical induction** to prove that

$m^3 + 2m$ is divisible by 3 for all integer $m \geq 1$.

See ver. A.

3. [5pt] Prove that any set $S \subseteq \{1, \dots, 120\}$ with $|S| = 61$ contains two numbers a and b such that a is divisible by b .

See ver. A .

$$140 \rightarrow 120$$

$$71 \rightarrow 61$$

4. [5pt] Let $m = 2021$ and $n = 110$. Find integers a and b such that

$$am + bn = 1.$$

$$2021 \div 110 = 18 \dots 41$$

$$110 \div 41 = 2 \dots 28$$

$$41 \div 28 = 1 \dots 13$$

$$28 \div 13 = 2 \dots 2$$

$$13 \div 2 = 6 \dots 1$$

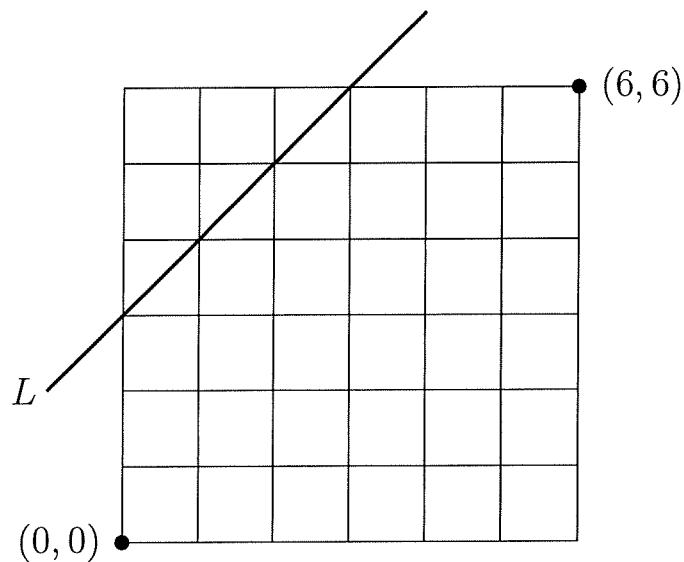
2021	110	
1	0	2021
0	1	110
1	-18	41
-2	37	13 28
3	-55	13
-8	147	2
51	-937	1

Double check:

$$51 \cdot 2021 + (-937) \cdot 110 = 1$$

$$\Rightarrow \underline{\underline{a = 51, b = -937}}$$

5. [extra 2pt] Consider two possible moves \rightarrow : $(1, 0)$ and \uparrow : $(0, 1)$. Count the number of ways to go from $(0, 0)$ to $(6, 6)$ such that
- each step is either \rightarrow or \uparrow , and
 - it **touches** the line $L : y = x + 3$. (有碰到就算)



See ver. A.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	