國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

離散數學（一） **MATH 203: Discrete Mathematics I**

第二次期中考 November 24, 2020 **Midterm 2**

姓名 Name : solution

學號 Student ID # : ____________

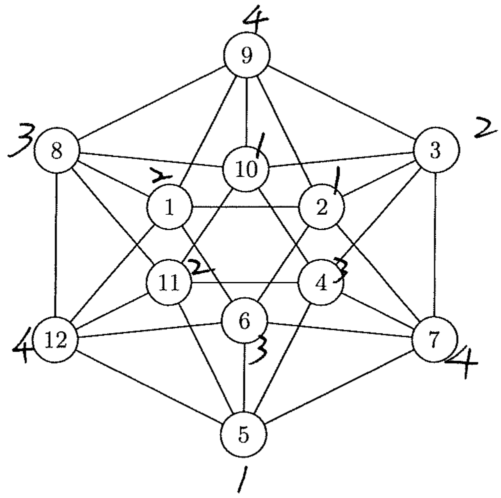
Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. Let G be the graph below. Answer the following questions and **provide your reasons**. [Hint: This graph is composed of the vertices and the edges of an icosahedron (正二十面體).]



(a) [1pt] Is there an Eulerian circuit on G?

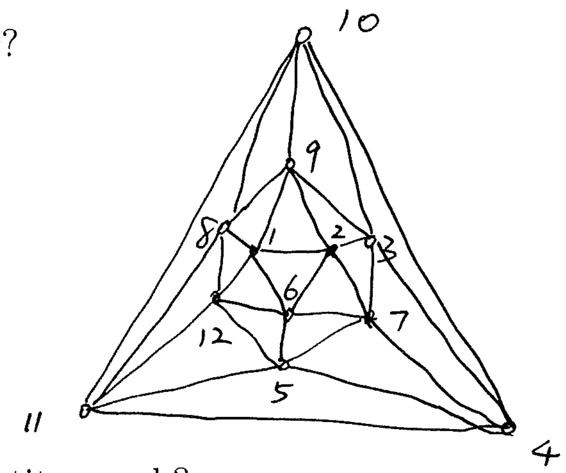
No, every vertex has degree 3, which is odd.

(b) [1pt] Is there a Hamiltonian cycle on G?

Yes, e.g., 9-8-12-5-6-1-2-3-7-4-11-10-9.

(c) [1pt] Is G planar?

Yes.



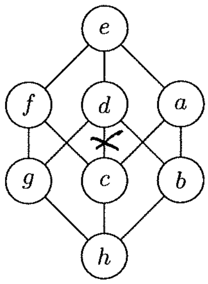
(d) [1pt] Is G a bipartite graph?

No, G ~~has~~ contains a triangle.

(e) [1pt] Is G 4-colorable?

Yes, e.g. ~~see~~ See above.

2. Let (X, R) be the poset with the Hasse diagram below. Answer the following questions and **provide your reasons.**

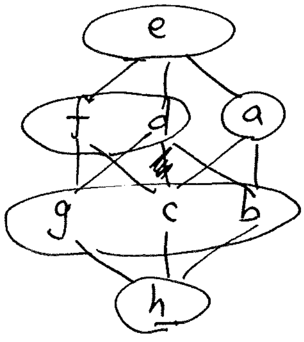


(a) [1pt] Is there a chain cover of (X, R) of size 2?

No, there is an antichain $\{f, d, a\}$ of size 3.

(b) [1pt] Is there an antichain cover of (X, R) of size 5?

Yes, e.g.



(c) [1pt] Find a linear extension of (X, R).

$h \leq g \leq c \leq b \leq f \leq d \leq a \leq e$.

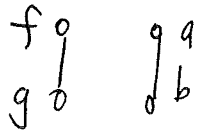
(d) [1pt] Find a total order on $\{a, \ldots, h\}$ that is not a linear extension of (X, R).

$a \leq b \leq c \leq \cdots \leq h$

since $a \geq b$ in R.

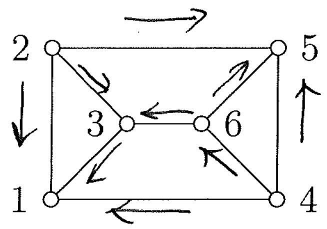
(e) [1pt] Is (X, R) an interval poset?

No, since ~~D(f) = {g, c, h} and D(a) =~~



is $2+2$.

3. [5pt] Determine whether the graph below is a comparability graph or not and **provide your reasons**.



Assume $2 \to 5 \Rightarrow 2 \to 1$, $2 \to 3$, $6 \to 5$, $4 \to 5$

$2 \to 3 \Rightarrow 6 \to 3$

$2 \to 1 \Rightarrow 4 \to 1$

$4 \to 1 \Rightarrow 3 \to 1$, $4 \to 6$

But then $3 \to 1 \Rightarrow 3 \to 6$

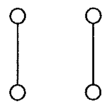
$\times$.

So the graph is not a comparability graph.

4. [5pt] Let (X, R) be a poset. Recall that

$$D(x) = \{y \in X : y \preceq x \text{ in } R\}$$

for any $x \in X$ and $\mathbf{2} + \mathbf{2}$ is the poset whose Hasse diagram is as below.



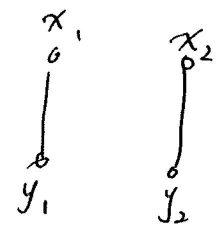
Show that the following statements are equivalent:

(a) (X, R) contains $\mathbf{2} + \mathbf{2}$ as a subposet.

(b) There are two elements $x_1, x_2 \in X$ such that $D(x_1) \setminus D(x_2) \neq \emptyset$ and $D(x_2) \setminus D(x_1) \neq \emptyset$.

(a) $\Rightarrow$ (b) Suppose (X,R) contains $2+2$.

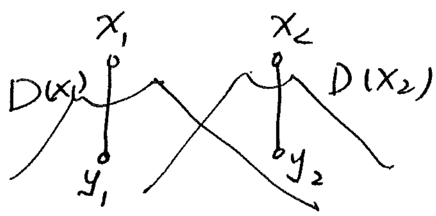
Then there are 4 points in X x_1, x_2, y_1, y_2



such that $x_1 \geq y_1$
$x_2 \geq y_2$
and $x_1 \parallel x_2$, $x_1 \parallel y_2$, ~~x~~ $y_1 \parallel x_2$, $y_1 \parallel y_2$.

Thus, $y_1 \in D(x_1) \setminus D(x_2)$
$y_2 \in D(x_2) \setminus D(x_1)$.

(b) $\Rightarrow$ (a). Pick $y_1 \in D(x_1) \setminus D(x_2)$



$y_2 \in D(x_2) \setminus D(x_1)$.

By def of $D(x_1)$ and $D(x_2)$,
$x_1 \geq y_1$ and $x_2 \geq y_2$.

If $x_1 \geq x_2$, then $D(x_2) \subseteq D(x_1)$.
If $x_1 \leq x_2$, then $D(x_1) \subseteq D(x_2)$. $\Rightarrow x_1 \parallel x_2$.

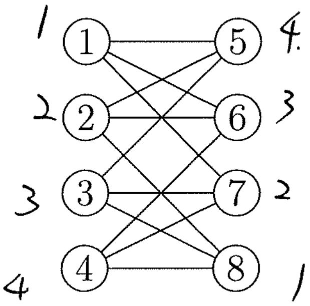
If $x_1 \geq y_2$, then $y_2 \in D(x_1)$
If $y_2 \geq x_1$, then ~~$D(x_2) \geq$~~ $D(x_1) \subseteq D(x_2)$ $\Rightarrow x_1 \parallel y_2$.

By Symmetry, $x_2 \parallel y_1$.

If $y_1 \leq y_2$, then $y_1 \in D(x_2)$ $\Rightarrow y_1 \parallel y_2$.
If $y_2 \leq y_1$, then $y_2 \in D(x_1)$

So x_1, x_2, y_1, y_2 induce a $2+2$ subposet.

5. [extra 2pt] Let G be the graph below. Find an order of the vertices (e.g., $8, 7, \ldots, 1$) such that the greedy coloring algorithm using this order needs 4 colors.



greedy coloring.

order: 1, 8, 2, 7, 3, 6, 4, 5.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	