國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

**離散數學（一）** **MATH 203: Discrete Mathematics I**

**第二次期中考** November 24, 2020 **Midterm 2**

姓名 Name : _____

學號 Student ID # : _____

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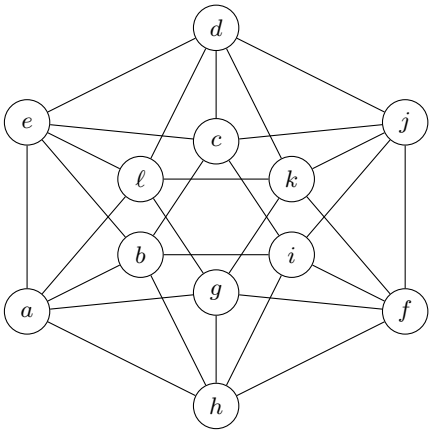
|---|---|
| Lecturer: | Jephian Lin 林晉宏 |
| Contents: | cover page, **5 pages** of questions, score page at the end |
| To be answered: | on the test paper |
| Duration: | **110 minutes** |
| Total points: | **20 points** + 2 extra points |

**Do not open this packet until instructed to do so.**

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. Let $G$ be the graph below. Answer the following questions and **provide your reasons**. [Hint: This graph is composed of the vertices and the edges of an icosahedron (正二十面體).]




(a) [1pt] Is $G$ a bipartite graph?

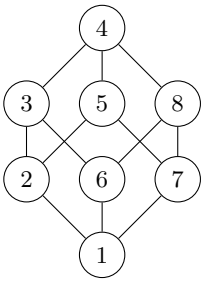
(b) [1pt] Is there an Eulerian circuit on $G$?

(c) [1pt] Is there a Hamiltonian cycle on $G$?

(d) [1pt] Is $G$ planar?

(e) [1pt] Is $G$ 4-colorable?

2. Let $(X, R)$ be the poset with the Hasse diagram below. Answer the following questions and **provide your reasons**.




(a) [1pt] Find a linear extension of $(X, R)$.

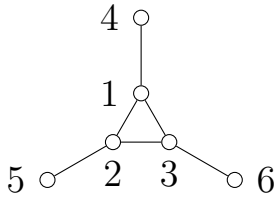
(b) [1pt] Find a total order on $\{1, \ldots, 8\}$ that is not a linear extension of $(X, R)$.

(c) [1pt] Is there a chain cover of $(X, R)$ of size 2?

(d) [1pt] Is there an antichain cover of $(X, R)$ of size 5?

(e) [1pt] Is $(X, R)$ an interval poset?

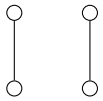
3. [5pt] Determine whether the graph below is a comparability graph or not and **provide your reasons**.



4. [5pt] Let $(X, R)$ be a poset. Recall that

$$D(x) = \{y \in X : y \preceq x \text{ in } R, y \neq x\}$$

for any $x \in X$ and $\mathbf{2} + \mathbf{2}$ is the poset whose Hasse diagram is as below.

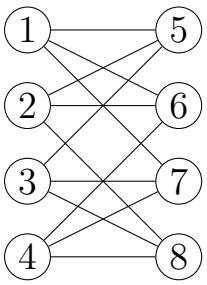


Show that the following statements are equivalent:

(a) $(X, R)$ contains $\mathbf{2} + \mathbf{2}$ as a subposet.

(b) There are two elements $x_1, x_2 \in X$ such that $D(x_1) \setminus D(x_2) \neq \emptyset$ and $D(x_2) \setminus D(x_1) \neq \emptyset$.

5. [extra 2pt] Let $G$ be the graph below. Find an order of the vertices (e.g., $8, 7, \ldots, 1$) such that the greedy coloring algorithm using this order needs 4 colors.




**[END]**

| Page | Points | Score |
|---|---|---|
| 1 | 5 | |
| 2 | 5 | |
| 3 | 5 | |
| 4 | 5 | |
| 5 | 2 | |
| Total | 20 (+2) | |