Math589 Homework 10

1. [1pt] Let G be a graph with a cut vertex v, whose removal results in a partition $V(G) - v = X \cup Y$ such that there is no edges between X and Y. Let $G_1 = G[X \cup \{v\}]$ and $G_2 = G[Y \cup \{v\}]$. Show that $\mathcal{C}(G)$ is the direct product of $\mathcal{C}(G_1)$ and $\mathcal{C}(G_2)$.

   Solution.

2. [1pt] Let G be a graph and e an edge of G. Let G_e be the subdivision of G at edge e. Show that $\mathcal{C}(G)$ has a sparse basis if and only if $\mathcal{C}(G_e)$ has a sparse basis.

Solution.

Questions to ponder:

1. Let G be the complete subdivision of K_5 (subdivide every edge exactly once). Use the counting argument to show that $\mathcal{C}(G)$ has no sparse basis.
2. Let G be the complete subdivision of $K_{3,3}$ (subdivide every edge exactly once). Use the counting argument to show that $\mathcal{C}(G)$ has no sparse basis.
3. Let G be the Petersen graph. Use the counting arguement to show that $\mathcal{C}(G)$ has no sparse basis.
4. Practice your TEXnique at `https://texnique.xyz/`.