Math589 Homework 11

1. [1pt] Let G be a graph and $\mathcal{S}(G)$ the family of matrices associated with G. Recall that the maximum nullity of G is

$$M(G) = \max\{\text{null}(A) : A \in \mathcal{S}(G)\}.$$

Now we define the maximum multiplicity of G as

$$M_e(G) = \max\{\text{mul}_\lambda(A) : A \in \mathcal{S}(G), \lambda \in \text{spec}(A)\}.$$

Here $\text{null}(A)$ is the nullity of A and $\text{mul}_\lambda(A)$ is the multiplicity of λ as an eigenvalue of A. Show that $M(G) = M_e(G)$ for every graph G.

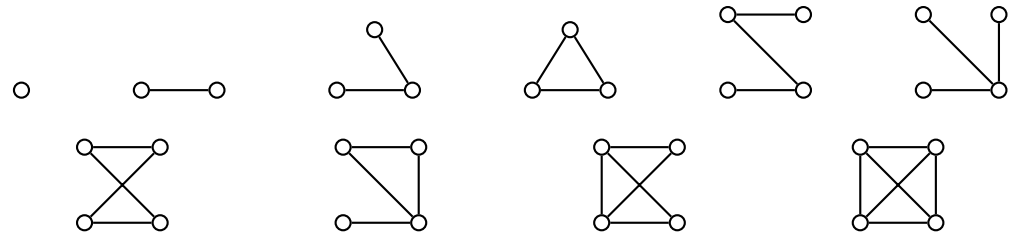
Solution.

2. [1pt] Characterize the graphs G with $M(G) = n$ and the graphs with $M(G) = n-1$, where n is the number of vertices.

   Solution.

Questions to ponder:

1. Pick a tree on at least 10 vertices. Find its zero forcing number.
2. Compute the maximum nullity and the zero forcing number of the following graphs.



3. Find a graph G with $P(G) < M(G)$, where $P(G)$ is the path cover number.
4. Find a graph G with $M(G) < P(G)$, where $P(G)$ is the path cover number.
5. Practice your TEXnique at `https://texnique.xyz/`.