

Math589 Homework 13

1. [1pt] Let $G = P_3$. There are 8 loop configurations on P_3 . (Each vertex can have a loop or have no loop on it.) For each of them, find its zero forcing number.

Solution.

2. [1pt] Let

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Is A a positive semidefinite matrix? If yes, prove it; if no, find a vector $\mathbf{x} \in \mathbb{R}^3$ such that $\mathbf{x}^\top A \mathbf{x} < 0$.

Solution.

Questions to ponder:

1. Let $G = P_n$. Which configuration \mathfrak{G} of G has $Z(\mathfrak{G}) = 0$?
2. Let $G = K_{1,n-1}$. Which configuration \mathfrak{G} of G has $Z(\mathfrak{G}) = 0$?
3. Use SageMath to find a graph G with $\widehat{Z}(G) \leq Z(G)$.
4. Find a graph G and its vertex v such that $Z(G) - Z(G - v) = 1$. Do the same for $Z(G) - Z(G - v) = 0$ or -1 .
5. Find a graph G and its edge e such that $Z(G) - Z(G - e) = 1$. Do the same for $Z(G) - Z(G - e) = 0$ or -1 .
6. Practice your \TeX nique at <https://texnique.xyz/>.