Math589 Homework 14

1. [1pt] Find a matrix $A \in \mathcal{S}(P_3)$ such that $\text{spec}(A) = \{1, 3, 5\}$ and $\text{spec}(A(1)) = \{2, 4\}$.

   Solution.

2. [1pt] Let

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

Find a basis of $\text{span}\{I, A, A^2, A^3\}$. Then write A^4 as a linear combination of your basis.

Solution.

Questions to ponder:

1. Find a 2×2 real symmetric matrix whose spectrum is $\{1, 3\}$.
2. Let $A = \begin{bmatrix} x & z \\ z & y \end{bmatrix}$. Find equations on x, y, z such that A has the spectrum $\{1, 3\}$. Can you draw the solutions of the equations on the 3-dimensional space? Can you parametrize the curver?
3. Let $f(x, y, z) = x^2 + y^2 + z^2$. Find $\frac{df}{dx}$.
4. Let $f(x, y, z) = (xy, yz, zx)$. Find $\frac{df}{d(x,y,z)}$.
5. Determine whether the unit sphere $x^2 + y^2 + z^2 = 1$ and the plane $x + y + z = 0$ intersect transversally at the point $(1, 0, 0)$.
6. Determine whether the unit sphere $x^2 + y^2 + z^2 = 1$ and the plane $x + y + z = \frac{3}{\sqrt{3}}$ intersect transversally at the point $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$.
7. Practice your TEXnique at `https://texnique.xyz/`.