

Math589 Homework 2

1. [1pt] A graph parameter $\xi(G)$ is called minor monotone if $\xi(G) \leq \xi(H)$ whenever G is a minor of H . Let $n(G)$ be the number of vertices of G , $m(G)$ the number of edges of G , and $c(G)$ the number of components of G . For what A, B, C is the linear combination

$$(An + Bm + Cc)(G) := An(G) + Bm(G) + Cc(G)$$

minor monotone?

Solution.

2. [1pt] Let

$$X = \{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\| = 1\}$$

be the unit circle. Show that X separates \mathbb{R}^2 . That is, suppose there is a curve (continuous function) $\phi : [0, 1] \rightarrow \mathbb{P} \subseteq \mathbb{R}^2$ such that $\phi(0)$ is inside the circle and $\phi(1)$ is outside the circle, then there must be a point $\phi(c)$, $c \in (0, 1)$, that is on the circle X .

Solution.

Questions to ponder:

1. Draw $K_{3,5}$ on a torus.
2. Show that the y -axis separates \mathbb{R}^2 .
3. Show that any triangle separates \mathbb{R}^2 .
4. Let $X \subseteq \mathbb{R}^2$ be a compact set. Show that each equivalence class of $\mathbb{R}^2 \setminus X$ under the relation $\mathbf{a} \sim \mathbf{b}$ if there is a polygonal arc in $\mathbb{R}^2 \setminus X$ connecting \mathbf{a} and \mathbf{b} is open.
5. Practice your \TeX nique at <https://texnique.xyz/>.