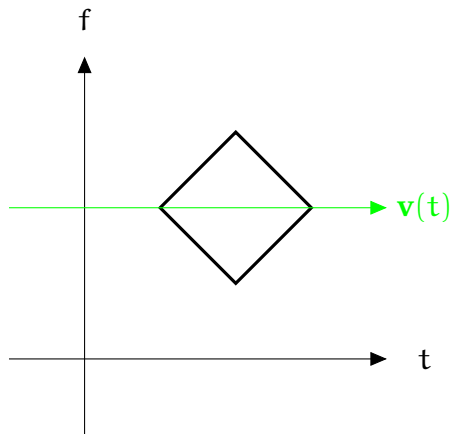


Math589 Homework 3

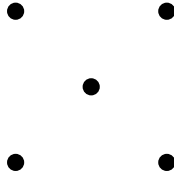
1. [1pt] Let X be the polygon shown as below. Let $\mathbf{v}(t) = (t, 2)$. Define a function $m : \mathbb{R}^2 \setminus X \rightarrow \mathbb{Z}$ such that $m(\mathbf{p})$ is the number of points in which the upward vertical ray from \mathbf{p} meets X . Consider the function $f(t) = m(\mathbf{v}(t))$. Use the blue color to draw the function of $f(t)$ on the same picture of X . (Put a hollow circle if f is not defined at some point.)

Solution.



2. [1pt] Use the picture below to draw as many non-crossing arcs as possible to between the five given vertices.

Solution.



Questions to ponder:

1. Let G be a plane graph and f a face of G . Let $H \subseteq G$ be a subgraph of G .
 - (a) Show that H has a face f' containing f .
 - (b) Show that if the frontier of f lies in H , then $f' = f$.
2. Visit the website https://radokirov.com/graph-editor.js/#graph_ed. Copy the JSON data below to the box with a button named "Parse JSON Data" and click on the button. Find a $K_{3,3}$ or a K_5 minor in it.

```
{"vertices": ["0", "1", "2", "3", "4", "5", "6", "7", "8", "9"],
  "edges": [["0", "5", null], ["0", "8", null], ["1", "2", null],
  ["1", "5", null], ["1", "6", null], ["1", "8", null], ["1", "9", null],
  ["2", "4", null], ["2", "5", null], ["3", "4", null], ["3", "9", null],
  ["4", "5", null], ["4", "7", null], ["4", "8", null], ["4", "9", null],
  ["5", "7", null], ["5", "8", null], ["7", "9", null]],
  "pos": null, "name": "G"}
```

3. Do the same with the next one.

```
{"vertices": ["0", "1", "2", "3", "4", "5", "6", "7", "8", "9"],
  "edges": [["0", "3", null], ["0", "4", null], ["0", "6", null],
  ["0", "7", null], ["0", "9", null], ["1", "2", null], ["1", "5", null],
  ["1", "6", null], ["1", "9", null], ["2", "4", null], ["4", "8", null],
  ["5", "7", null], ["5", "8", null], ["6", "7", null], ["6", "8", null],
  ["7", "8", null], ["7", "9", null]],
  "pos": null, "name": "G"}
```

4. Practice your \TeX nique at <https://texnique.xyz/>.