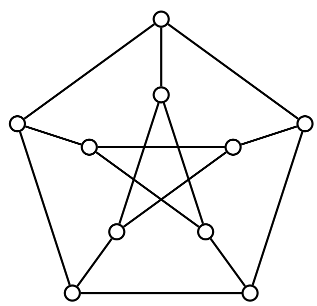
Math589 Homework 5

1. [1pt] Let G be the Petersen graph. Read the proof of Proposition 3.1.1 in the textbook and obtain an "ear decomposition" $(H_1, P_1), \ldots, (H_m, P_m)$ of G such that

   - H_1 is a cycle.
   - P_i is a path connecting two distinct vertices of H_i. (possibly just an edge)
   - $H_{i+1} = H_i \cup P_i$.
   - $G = H_m \cup P_m$.

   Use different colors to described your H_1 and P_i's.

   Solution.



2. [1pt] Find a connected plane graph G_1 and a face f_1 of G_1 such that the boundary of f_1 is not a cycle. Find a 2-connected plan graph G_2 and a face f_2 of G_2 such that the boundary of f_2 is not a non-separating cycle.

Solution.

Questions to ponder:

1. Pick a graph that is 2-connected and two vertices x and y on it. Find two internal vertex-disjoint paths connecting x and y.
2. Pick a graph that is 3-connected and two vertices x and y on it. Find three internal vertex-disjoint paths connecting x and y.
3. Is the Petersen graph 2-connected or 3-connected?
4. Present the proof of Proposition 3.1.1 (in your own words). You are encouraged to write down the proof first.
5. Practice your TEXnique at `https://texnique.xyz/`.
6. Let G be a graph. Google how to use SageMath to test if G is planar or not; moreover, use SageMath to draw G on $\mathbb{R}^2$ without crossing. You may use SageCell to try your code.