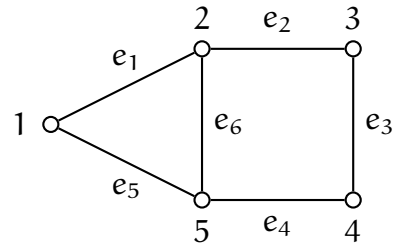
Math589 Homework 8

1. [1pt] Let G be the graph with labeled vertices and edges as shown below. Find all elements in the cycle space $\mathcal{C}(G)$.



Solution.

2. [1pt] Let G be the graph with labeled vertices and edges as in the previous question. Find all elements in the cut space $\mathcal{B}(G)$ and mark each bond.

   Solution.

Questions to ponder:

1. Let 0 and 1 be the elements in $\mathbb{F}_2$. Finish the following tables for addition and multiplication.

$+$	0	1
0		
1		

$\times$	0	1
0		
1		

2. Let G be a graph. Recall that $E(v)$ is the cut between $\{v\}$ and $V(G) \setminus \{v\}$. Is $E(v)$ always a bond? Is a bond always $E(v)$ for some v?

3. Let G be the graph as in Page 1. Let T be the spanning tree whose edges are $E(G) \setminus \{e_4, e_5\}$. Write the cycle $\{e_1, e_2, e_3, e_4, e_5\}$ as a sum of fundamental cycles. Write the cut $\{e_1, e_4, e_6\}$ as a sum of fundamental cuts.

4. Let
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$
be a matrix in $\mathbb{F}_2$. Find a basis of its kernel and a basis of its row space.

5. Practice your TEXnique at `https://texnique.xyz/`.