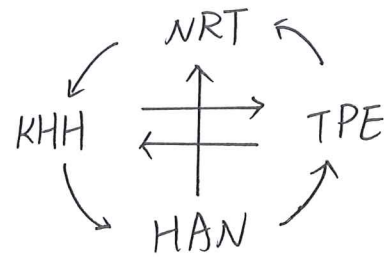


Graph theory : basics.

一組機票亂了，排得回來嗎？(找一下起點 & 終點)

KHH \rightarrow TPE HAN \rightarrow NRT
TPE \rightarrow KHH NRT \rightarrow KHH
TPE \rightarrow NRT HAN \rightarrow TPE
KHH \rightarrow HAN



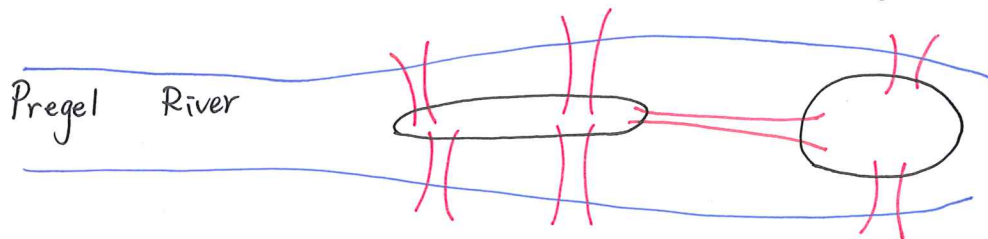
\Rightarrow 起 : HAN , 終 : NRT

七橋問題 : 能否每座橋各走一次，回到原點？

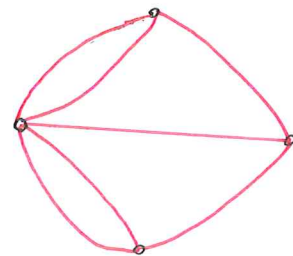
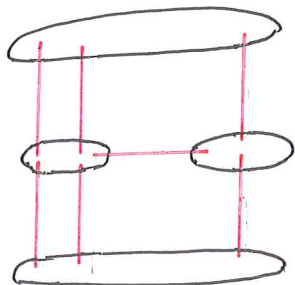
Leonhard Euler
(1707 - 1783)

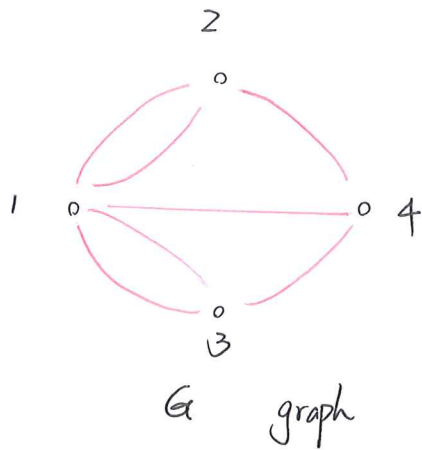
Königsberg, Prussia

(now Kaliningrad, Russia)



一筆畫問題 :





Vertices ^點 $V(G) = \{1, 2, 3, 4\}$
(or nodes)

edges ^邊 $E(G) = \{12, 12, 13, 13, 14, 24, 34, 34\}$

• two vertices i, j are adjacent if $ij \in E(G)$,

^{相鄰}

• the neighbors of vertex i is

^{鄰居}

$$N(i) = \{j : ij \in E(G)\}$$

e.g. $N(2) = \{1, 4\}$

• the degree of vertex i is

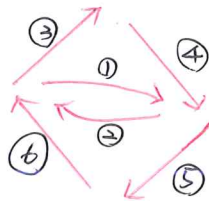
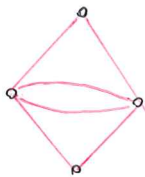
^{度數}

$\deg(i) = \text{number of edges on } i$

e.g. $\deg(1) = 5$

• an Eulerian circuit is a circuit using all edges.

e.g.



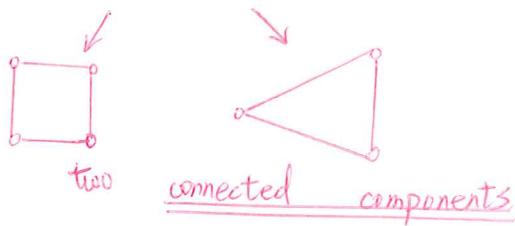
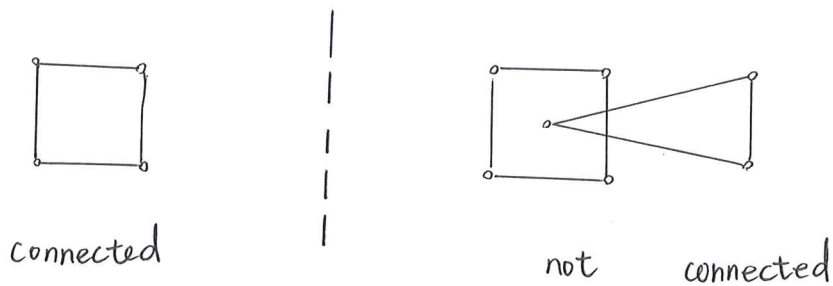
Thm (Euler 1736)

A graph G has an Eulerian circuit

if and only if

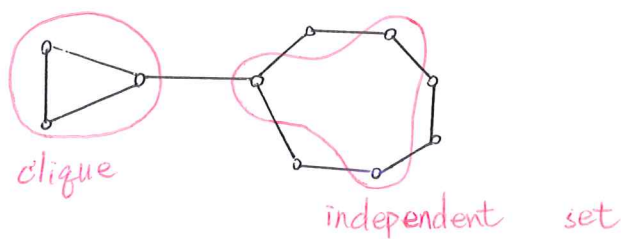
$\left\{ \begin{array}{l} G \text{ is } \underline{\text{connected}} \text{ and} \\ \text{every vertex has even degree.} \end{array} \right.$

- A graph is connected if for any two vertices i, j , one may "walk" from i and j .



Q: How to test if a graph is connected?

- A clique is a set of vertices such that any two vertices are adjacent.
- A independent set is a set of vertices such that any two vertices are not adjacent.



- A bipartite graph is a graph such that $V(G)$ has a partition (X, Y) and X, Y are independent sets.

