國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（二） MATH 104 / GEAI 1209: Linear Algebra II

期末考 June 8, 2020 **Final Exam**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **8 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**30 points** + 7 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. [1pt] Give an example of a 2×2 matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ such that A is an orthogonal matrix (i.e., $A^\top A = I$) with $a_{12} \neq 0$.

2. [1pt] Give an example of a 2×2 matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ such that A is diagonalizable and $a_{12} \neq 0$.

3. [1pt] Give an example of a 2×2 matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ such that A is not diagonalizable and $a_{12} \neq 0$.

4. [1pt] Find a 2×2 matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ such that the eigenvalues of A are $\{1, 5\}$ and $a_{12} = a_{21} = 2$.

5. [1pt] Give an example of a 5×5 matrix A whose only eigenvalue is 3 with algebraic multiplicity 5 and geometric multiplicity 2.

6. Let E_{ij} be the 2×3 matrix whose entries are all zeros except that the i, j-entry is one. Then

$$\mathcal{B} = \{E_{11}, E_{12}, E_{13}, E_{21}, E_{22}, E_{23}\}$$

is a basis of $\mathcal{M}_{2\times 3}$, the space of all 2×3 real matrices. Suppose $f : \mathcal{M}_{2\times 3} \to \mathcal{M}_{2\times 3}$ is a homomorphism such that $\text{Rep}_{\mathcal{B},\mathcal{B}}(f)$ equals

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

(a) [1pt] Let $M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. Find $f(M)$.

(b) [2pt] Find the range of f.

(c) [2pt] Find the nullspace of f.

7. [5pt] Let

$$A = \begin{bmatrix} 3 & 8 & -24 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{bmatrix}.$$

Find an invertible matrix Q and a diagonal matrix D such that $AQ = QD$.

8. [5pt] Let

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and

$$p(x) = \det(A - xI) = a_0x^6 + a_1x^5 + a_2x^4 + a_3x^3 + a_4x^2 + a_5x + a_6$$

its characteristic polynomial. Find a_0, a_1, a_2, a_5, and a_6.

9. [5pt] Let J_n be the $n \times n$ all-ones matrix. Let I_n be the $n \times n$ identity matrix. Find $\det(J_n + I_n)$ as a formula of n. Make sure to justify every step of your argument.

10. [5pt] Let U be an $n \times n$ real upper-triangular matrix. Show that if $UU^\top = U^\top U$, then U is a diagonal matrix.

11. [extra 5pt] Let

$$A = \begin{bmatrix} -3 & 10 \\ 1 & 0 \end{bmatrix}.$$

Find A^{100}. [Hint: Write A as QDQ^{-1}.]

12. [extra 2pt] Let

$$p(x) = x^3 + x^2 - 2x.$$

Find a 3×3 matrix A such that $p(A) = O$ and the three eigenvalues of A are all distinct.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	2	
Total	30 (+7)	