

Sample Questions 10

For Problems 1~3, let $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

solution for $(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = \mathbf{0}$.

1. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \text{ and } \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Find λ_1 and λ_2 such that $\mathbf{A}\mathbf{v}_1 = \lambda_1\mathbf{v}_1$ and $\mathbf{A}\mathbf{v}_2 = \lambda_2\mathbf{v}_2$. Also, find an invertible matrix \mathbf{Q} and a diagonal matrix \mathbf{D} such that $\mathbf{A}\mathbf{Q} = \mathbf{Q}\mathbf{D}$.

2. Let $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Compute $p(x) = \det(\mathbf{A} - x\mathbf{I})$ and then solve the equation $p(x) = 0$.

3. For each of $\lambda = 0, 2$, find a nonzero

4. Diagonalize $\mathbf{A} = \begin{bmatrix} -2 & 15 \\ 1 & 0 \end{bmatrix}$.

5. Diagonalize $\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

6. Diagonalize $\mathbf{A} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

7. Diagonalize $\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.