Sample Questions 4

1. Let V be a two-dimensional space in $\mathbb{R}^3$ and f is the (orthogonal) projection map onto V. Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis of $\mathbb{R}^3$ such that $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis of V and $\mathbf{v}_3$ is orthogonal to any vector on V. Find $\mathrm{Rep}_{\mathcal{B},\mathcal{B}}(f)$.

2. Let V, $\mathcal{B}$ be as the previous question. Let g be the reflection map with respect to the plane V. That is, treating V as a mirror. Find $\mathrm{Rep}_{\mathcal{B},\mathcal{B}}(f)$.

3. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the unit vector in $\mathbb{R}^2$ with angles $\frac{\pi}{4}$ and $\frac{3\pi}{4}$. Any vector $\mathbf{v} \in \mathbb{R}^2$ can be written as $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$ for some coefficients c_1 and c_2. Define the scaling map $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that $f(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) = 2c_1\mathbf{v}_1 + c_2\mathbf{v}_2$. Find $\mathrm{Rep}_{\mathcal{B},\mathcal{B}}(f)$ and $\mathrm{Rep}_{\mathcal{E},\mathcal{E}}(f)$, where $\mathcal{E}$ is the standard basis of $\mathbb{R}^2$.

4. Let $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$ be a basis of a space V. Suppose $f : V \to V$ is a map such that

$$\mathrm{Rep}_{\mathcal{B},\mathcal{B}}(f) = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

   Find $f(\mathbf{v}_1 + \mathbf{v}_3)$.

5. Let $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$ and $\mathcal{D} = \{\mathbf{u}_1, \ldots, \mathbf{u}_3\}$ be bases of the spaces V and W, respectively. Suppose $f : V \to W$ is a map such that $f(\mathbf{v}_1) = 5\mathbf{u}_1$, $f(\mathbf{v}_2) = 3\mathbf{v}_2$, and $f(\mathbf{v}_3) = f(\mathbf{v}_4) = \mathbf{0}$. Find $\mathrm{Rep}_{\mathcal{B},\mathcal{D}}(f)$.

6. Let $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_4\}$ and $\mathcal{D} = \{\mathbf{u}_1, \ldots, \mathbf{u}_3\}$ be bases of the spaces V and W, respectively. Suppose $f : V \to W$ is a map such that

$$\mathrm{Rep}_{\mathcal{B},\mathcal{D}}(f) = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

   Find $f(\mathbf{v}_1 + \mathbf{v}_3)$.

7. Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

   and $f : \mathbb{R}^4 \to \mathbb{R}^4$ a map defined by $f(\mathbf{v}) = A\mathbf{v}$. Let B be a basis whose vectors are the columns of

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & -1 \end{bmatrix}.$$

   Find $\mathrm{Rep}_{\mathcal{B},\mathcal{B}}(f)$.