Sample Solutions for Sample Questions 4.

1. $f(\vec{v}_1) = \vec{v}_1$
$f(\vec{v}_2) = \vec{v}_2$
$f(\vec{v}_3) = \vec{0}$

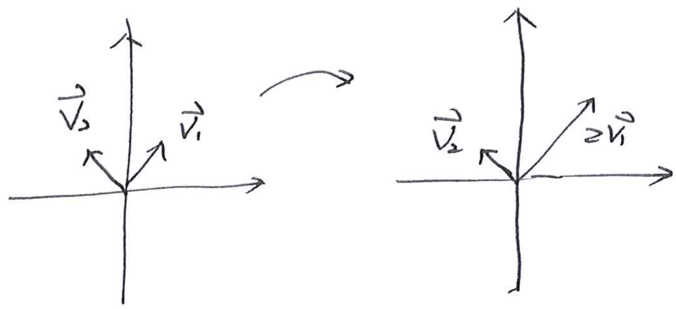
$$\xrightarrow{Rep_B} \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}. \Rightarrow Rep_{B,B}(f) = \begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}$$

2. $f(\vec{v}_1) = \vec{v}_1$
$f(\vec{v}_2) = \vec{v}_2$
$f(\vec{v}_3) = -\vec{v}_3$

$$\xrightarrow{Rep_B} \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\-1\end{pmatrix} \Rightarrow Rep_{B,B}(f) = \begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix}$$

3. $f(\vec{v}_1) = 2\vec{v}_1$
$f(\vec{v}_2) = \vec{v}_2$

$$\xrightarrow{Rep_B} \begin{pmatrix}2\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$$



$$\Rightarrow Rep_{B,B}(f) = \begin{pmatrix}2&0\\0&1\end{pmatrix}$$

To compute $Rep_{\mathcal{E},\mathcal{E}}(f)$, use $Rep_{\mathcal{E},\mathcal{E}}(f) = Rep_{B,\mathcal{E}}(id) \cdot Rep_{B,B}(f) \cdot Rep_{\mathcal{E},B}(id)$.

We can calculate $\vec{v}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, $\vec{v}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}$, so $Rep_{B,\mathcal{E}}(id) = \frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\\1&1\end{pmatrix}$

Then $Rep_{\mathcal{E},B}(id) = Rep_{B,\mathcal{E}}(id)^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\-1&1\end{pmatrix}$

$$\Rightarrow Rep_{\mathcal{E},\mathcal{E}}(f) = \frac{1}{\sqrt{2}}\begin{pmatrix}1&-1\\1&1\end{pmatrix}\begin{pmatrix}2&0\\0&1\end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\-1&1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}3&1\\1&3\end{pmatrix}.$$

4.

$$\begin{pmatrix} 3 & & & \\ & 2 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \begin{array}{c} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \end{array} \begin{array}{c} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{array} = \begin{pmatrix} 3 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{array}{c} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{array}$$

So. $f(\vec{v}_1 + \vec{v}_3) = 3\vec{v}_1 + \vec{v}_3$.

5. $\cancel{6}$

$$\begin{array}{lcc} f(\vec{v}_1) = 5\vec{u}_1 & & \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \\ f(\vec{v}_2) = 3\vec{u}_2 & \xrightarrow{Rep_D} & \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \\ f(\vec{v}_3) = \vec{0} & & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ f(\vec{v}_4) = \vec{0} & & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \end{array} \Rightarrow Rep_{B,D}(f) = \begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

6.

$$\begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{array}{c} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{array} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \begin{array}{c} \vec{u}_1 \\ \vec{u}_2 \\ \vec{u}_3 \end{array}$$

So $f(\vec{v}_1 + \vec{v}_3) = 5\vec{u}_1$

7.

$$f(\vec{v}_1) = A\vec{v}_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} = 2\vec{v}_1 \qquad \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$f(\vec{v}_2) = A\vec{v}_2 = \begin{pmatrix} -2 \\ 2 \\ -2 \\ 2 \end{pmatrix} = -2\vec{v}_2 \xrightarrow{Rep_B} \begin{pmatrix} 0 \\ -2 \\ 0 \\ 0 \end{pmatrix}$$

$$f(\vec{v}_3) = A\vec{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \vec{0} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$f(\vec{v}_4) = A\vec{v}_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \vec{0} \qquad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Longrightarrow \quad Rep_{B,B}(f) = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$