

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (一)

MATH 103 / GEAI 1215: Linear Algebra I

第一次期中考

November 1, 2021

Midterm 1

姓名 Name : _____

學號 Student ID # : _____

Lecturer: Jephian Lin 林晉宏

Contents: cover page,
5 pages of questions,
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \end{bmatrix} \text{ and } \mathbf{p} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

(a) [1pt] Find a vector in $\text{Row}(A)$ that is nowhere zero (每一項都不是零).

(b) [1pt] Find a vector in $\text{Col}(A)$ that is nowhere zero.

(c) [1pt] Find a vector in $\ker(A)$ that is nowhere zero.

(d) [1pt] Find a vector in $\mathbf{p} + \ker(A)$ that is nowhere zero.

(e) [1pt] Let

$$B = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a 3×3 matrix E such that $EA = B$.

2. Let

$$A = \begin{bmatrix} 1 & -2 & -2 & -2 \\ 3 & -6 & -6 & -5 \\ -11 & 22 & 22 & 19 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}.$$

(a) [2pt] Find the reduced row echelon form of the augmented matrix $[A \mid \mathbf{b}]$.

(b) [3pt] Find \mathbf{p} , \mathbf{h}_1 , \mathbf{h}_2 such that

$$\{\mathbf{x} \in \mathbb{R}^4 : A\mathbf{x} = \mathbf{b}\} = \mathbf{p} + \text{span}(\{\mathbf{h}_1, \mathbf{h}_2\}).$$

3. Let

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

(a) [3pt] Find \mathbf{w} and \mathbf{h} such that $\mathbf{b} = \mathbf{w} + \mathbf{h}$ with $\mathbf{w} \in \text{Col}(A)$ and $\mathbf{h} \in \text{Col}(A)^\perp$.

(b) [2pt] Let θ be the angle between \mathbf{b} and \mathbf{w} . Find $\cos \theta$.

4. [5pt] 數學作文：請寫一篇短文來向沒修過線性代數的朋友介紹什麼是子空間 (subspace)。

請以盡量白話的敘述、或是比喻來介紹什麼是子空間？為什麼要考慮這樣的概念？並給一些能幫助他人理解的例子（正面的、反面的）；有必要的話可以加上一些圖來輔助說明。格式沒有限制，篇幅大約半面到一面。

(If Chinese is not your native language, you may use English or the language that you prefer.)

5. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Find a 5×5 matrix E such that $EAE^T = I$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	