

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

離散數學 (二)

MATH 207: Discrete Mathematics II

期末考

June 22, 2021

Final Exam

姓名 Name : _____

學號 Student ID # : _____

Lecturer: Jephian Lin 林晉宏

Contents: cover page,
6 pages of questions,
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 7 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

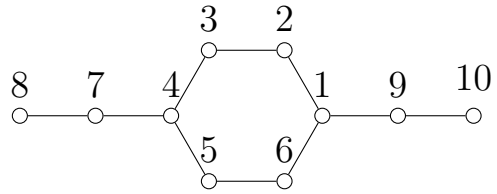
1. [5pt] Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

(a) [3pt] Find a permutation P such that $A = PBP^\top$.

(b) [2pt] Find the inertia (n_+, n_-, n_0) of A .

2. Let G be the graph below and A its adjacency matrix.



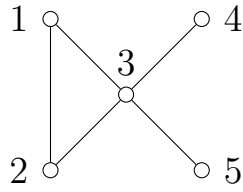
(a) [1pt] Find $\text{tr}(A^2)$.

(b) [1pt] Find $\text{tr}(A^3)$.

(c) [2pt] Draw all elementary subgraphs of G .

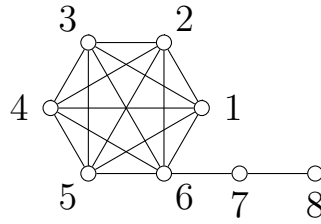
(d) [1pt] Find $\det(A)$.

3. [5pt] Let G be the graph below.



Let A be the adjacency matrix of G . Find $\det(A - xI)$, the characteristic polynomial of A .

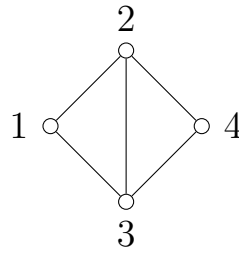
4. [5pt] Let G be the graph below.



Find the number of spanning trees on G .

5. [extra 5pt] Let P_{n+1} be the path on $n + 1$ vertices such that 1 is one of its endpoints. Let A be the adjacency matrix of P_{n+1} . Find the 1, 1-entry of A^{2n} .

6. [extra 2pt] Let G be the graph below.



Consider G as an electronic circuit such that each edge is a wire of resistance 1Ω . Find the effective resistance from 1 to 4.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	2	
Total	20 (+7)	