國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

離散數學（二） **MATH 207: Discrete Mathematics II**

期末考 June 22, 2021 **Final Exam**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	6 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 7 extra points

Do not open this packet until instructed to do so.

Instructions:

– Enter your **Name** and **Student ID #** before you start.

– Using the calculator is not allowed (and not necessary) for this exam.

– Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

– Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

– 可用中文或英文作答

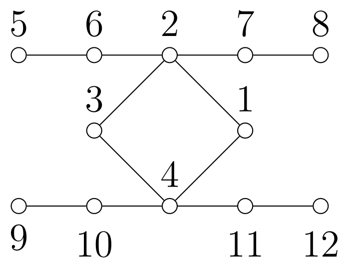
1. [5pt] Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

(a) [3pt] Find a permutation P such that $A = PBP^\top$.

(b) [2pt] Find the inertia (n_+, n_-, n_0) of A.

2. Let G be the graph below and A its adjacency matrix.



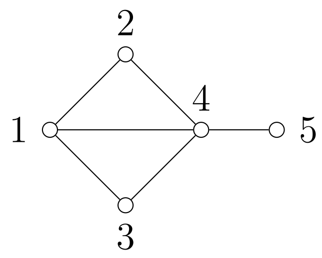
(a) [1pt] Find $\mathrm{tr}(A^2)$.

(b) [1pt] Find $\mathrm{tr}(A^3)$.

(c) [2pt] Draw all elementary subgraphs of G.

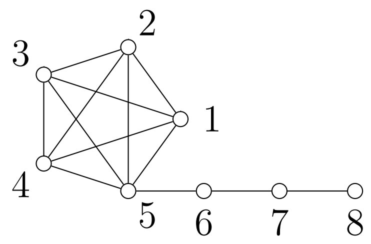
(d) [1pt] Find $\det(A)$.

3. [5pt] Let G be the graph below.



Let A be the adjacency matrix of G. Find $\det(A-xI)$, the characteristic polynomial of A.

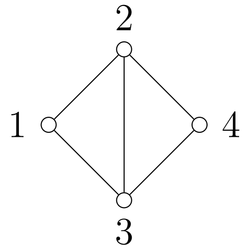
4. [5pt] Let G be the graph below.



Find the number of spanning trees on G.

5. [extra 5pt] Let P_{n+1} be the path on $n+1$ vertices such that 1 is one of its endpoints. Let A be the adjacency matrix of P_{n+1}. Find the $1,1$-entry of A^{2n}.

6. [extra 2pt] Let G be the graph below.



Consider G as an electronic circuit such that each edge is a wire of resistance 1Ω. Find the effective resistance from 1 to 4.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	2	
Total	20 (+7)	