國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

離散數學（二） **MATH 207: Discrete Mathematics II**

期末考 June 22, 2021 **Final Exam**

姓名 Name : solution

學號 Student ID # : ____________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	6 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 7 extra points

Do not open this packet until instructed to do so.

Instructions:

– Enter your **Name** and **Student ID #** before you start.

– Using the calculator is not allowed (and not necessary) for this exam.

– Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

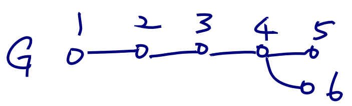
– Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

– 可用中文或英文作答

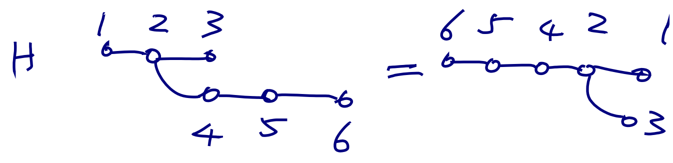
1. [5pt] Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

(a) [3pt] Find a permutation P such that $A = PBP^\top$.



$A = A(G)$



$B = A(H)$

H is labeled by permutation $\sigma = 654213$

(or 654231)

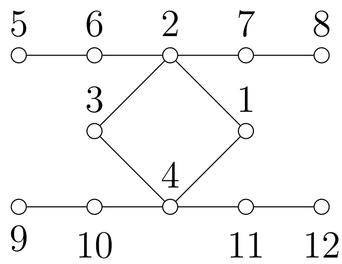
$$\Rightarrow P = \left(\begin{array}{ccc|ccc} 0 & & & & & 1 \\ & 0 & & & 1 & \\ & & 0 & 1 & & \\ \hline & 1 & & 0 & & \\ 1 & & & & 0 & \\ & & 1 & & & 0 \end{array}\right) \text{ or } \left(\begin{array}{ccc|ccc} 0 & & & & & 1 \\ & 0 & & & 1 & \\ & & 0 & 1 & & \\ \hline & 1 & & 0 & & \\ & & 1 & & 0 & \\ 1 & & & & & 0 \end{array}\right)$$

(b) [2pt] Find the inertia (n_+, n_-, n_0) of A.

$$A \rightsquigarrow \begin{pmatrix} 0 & 1 & & & & \\ 1 & 0 & & & & \\ & & 0 & 1 & & \\ & & 1 & 0 & 1 & 1 \\ & & & 1 & 0 & \\ & & & 1 & & 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 0 & 1 & & & & \\ 1 & 0 & & & & \\ & & 0 & 1 & & \\ & & 1 & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$$

$\Rightarrow$ inertia $= \underline{\underline{(2, 2, 2)}}$

2. Let G be the graph below and A its adjacency matrix.



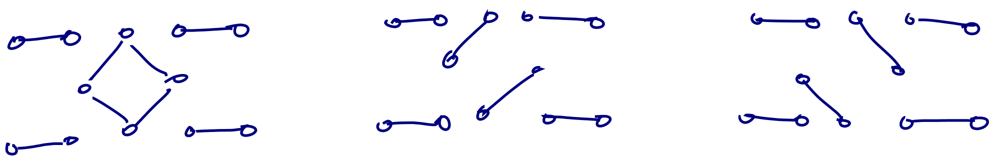
(a) [1pt] Find $\mathrm{tr}(A^2)$.

$\mathrm{tr}(A^2) = 2m = \underline{\underline{24}}$

(b) [1pt] Find $\mathrm{tr}(A^3)$.

$\mathrm{tr}(A^3) = 6t = \underline{\underline{0}}$

(c) [2pt] Draw all elementary subgraphs of G.



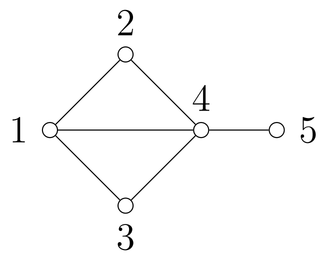
H_1 H_2 H_3

(d) [1pt] Find $\det(A)$.

	H_1	H_2	H_3
c	1	0	0
m	4	6	6
$(-1)^{n+c+m} 2^c$	-2	1	1

$\Rightarrow \det = -2+1+1 = \underline{0}$

3. [5pt] Let G be the graph below.



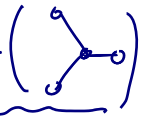
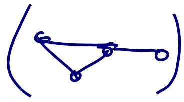
Let A be the adjacency matrix of G. Find $\det(A - xI)$, the characteristic polynomial of A.

$S_0 = 1$

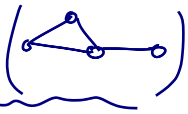
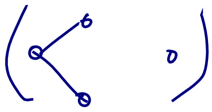
$S_1 = 0$

$S_2 = -m = -6$

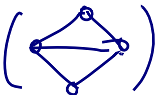
$S_3 = 2t = 4$

$S_4 = \det($  $)$ + $\det($  $)$

0 1

$+ \det($  $)$ + $\det($  $)$

1 0

$+ \det($  $)$

0

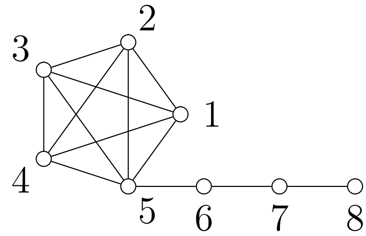
$= 2$

$S_5 = 0$ since no elementary subgraph

$\Rightarrow$ charpoly $= (-x)^5 + 0(-x)^4 - 6(-x)^3 + 4(-x)^2 + 2(-x)^1 + 0$

$= \underline{\underline{-x^5 + 6x^3 + 4x^2 - 2x}}$

4. [5pt] Let G be the graph below.



Find the number of spanning trees on G.

# of spanning tree on G

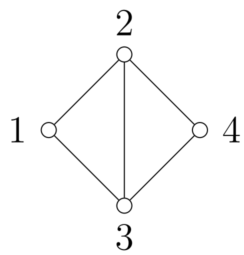
$= \#$ of spanning tree on K_5

$= \underline{5^3}$ by Calyey's formula

5. [extra 5pt] Let P_{n+1} be the path on $n+1$ vertices such that 1 is one of its endpoints. Let A be the adjacency matrix of P_{n+1}. Find the $1,1$-entry of A^{2n}.

See ver. A

6. [extra 2pt] Let G be the graph below.



Consider G as an electronic circuit such that each edge is a wire of resistance 1Ω. Find the effective resistance from 1 to 4.

See ver. A.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	2	
Total	20 (+7)	