

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

離散數學 (二)

MATH 207: Discrete Mathematics II

第二次期中考

May 11, 2021

Midterm 2

姓名 Name : _____

學號 Student ID # : _____

Lecturer: Jephian Lin 林晉宏

Contents: cover page,
5 pages of questions,
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

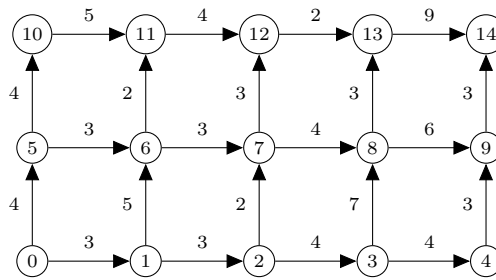
Total points: **20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. Let Γ be the directed graph as shown below. The number on each edge is its weight (distance).



(a) [1pt] What is the distance from 0 to 10?

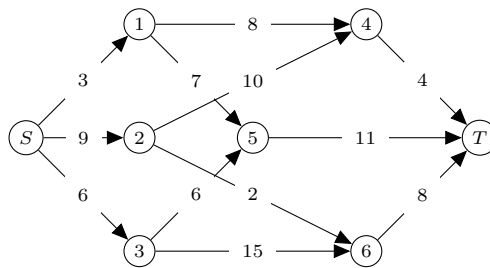
(b) [1pt] What is the distance from 0 to 11?

(c) [1pt] What is the distance from 0 to 12?

(d) [1pt] What is the distance from 0 to 13?

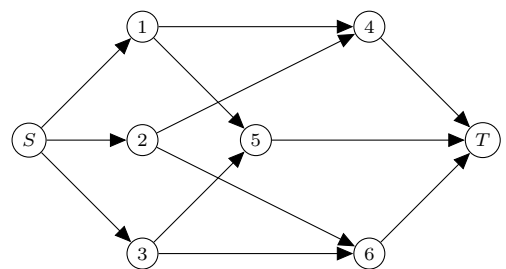
(e) [1pt] What is the distance from 0 to 14?

2. [5pt] Let Γ be the directed graph below, where S and T are the source and the sink, respectively. The number on each edge is its capacity.



Find a flow function f with the maximum value and a cut (A, B) with the minimum capacity.

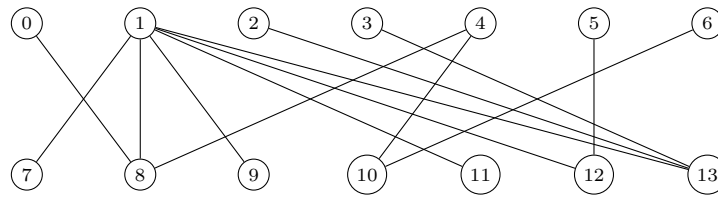
[Note: Use the graph at the bottom to answer your flow on each edge and your A and B .]



$A =$

$B =$

3. [5pt] Let G be the bipartite graph below. Find a maximum matching and a minimum vertex cover of G .



4. [5pt] Let (Γ, S, T, c) be a network on the directed graph Γ , where S , T , and c are the source, the sink, and the capacity function, respectively. Let f be a flow on this network and (A, B) a cut. Show that

$$\sum_{\substack{(u,v) \in E(\Gamma) \\ u \in A \\ v \in B}} f(u, v) - \sum_{\substack{(u,v) \in E(\Gamma) \\ u \in B \\ v \in A}} f(u, v) = \sum_{\substack{v \in V(\Gamma) \\ (S,v) \in E(\Gamma)}} f(S, v) - \sum_{\substack{u \in V(\Gamma) \\ (u,S) \in E(\Gamma)}} f(u, S).$$

5. [extra 2pt] Let $\mathbf{v}_1, \dots, \mathbf{v}_5$ be the columns of

$$\begin{bmatrix} 1 & 4 & 5 & 2 & 3 \\ 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 2 & 1 \end{bmatrix}.$$

Let $\mathbf{e}_1 = (1, 0, 0)^\top$. Find a subset $S \subseteq \{1, 2, 3, 4, 5\}$ such that $\{\mathbf{v}_i\}_{i \in S}$ is a basis and

$$\sum_{i \in S} \langle \mathbf{e}_1, \mathbf{v}_i \rangle$$

is minimized. Here $\langle \mathbf{x}, \mathbf{y} \rangle$ is the standard inner product in \mathbb{R}^3 .

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	