國立中山大學　　NATIONAL SUN YAT-SEN UNIVERSITY

離散數學（二）　　MATH 207: Discrete Mathematics II

第二次期中考　　May 11, 2021　　**Midterm 2**

姓名 Name　: ____________________

學號 Student ID #　: ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

– Enter your **Name** and **Student ID #** before you start.

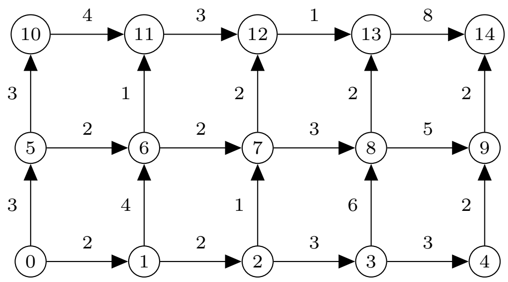
– Using the calculator is not allowed (and not necessary) for this exam.

– Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

– Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

– 可用中文或英文作答

1. Let Γ be the directed graph as shown below. The number on each edge is its weight (distance).



(a) [1pt] What is the distance from 0 to 10?

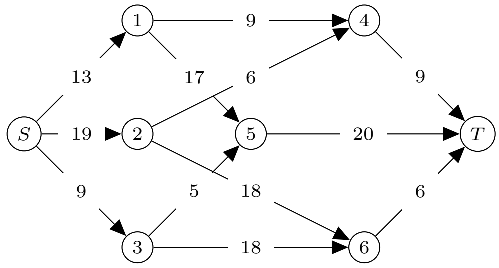
(b) [1pt] What is the distance from 0 to 11?

(c) [1pt] What is the distance from 0 to 12?

(d) [1pt] What is the distance from 0 to 13?

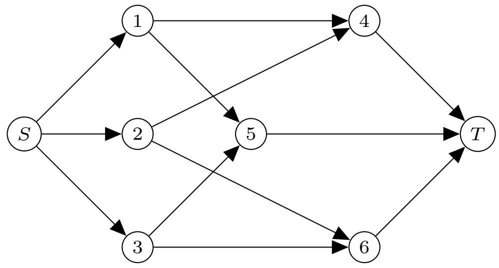
(e) [1pt] What is the distance from 0 to 14?

2. [5pt] Let Γ be the directed graph below, where S and T are the source and the sink, respectively. The number on each edge is its capacity.



Find a flow function f with the maximum value and a cut (A, B) with the minimum capacity.

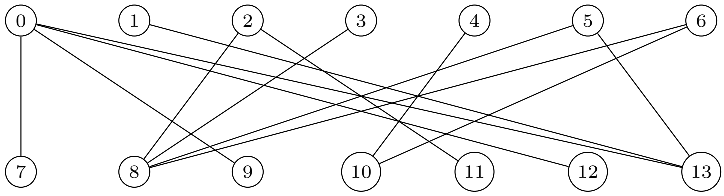
[Note: Use the graph at the bottom to answer your flow on each edge and your A and B.]



$A =$

$B =$

3. [5pt] Let G be the bipartite graph below. Find a maximum matching and a minimum vertex cover of G.



4. [5pt] Let (Γ, S, T, c) be a network on the directed graph Γ, where S, T, and c are the source, the sink, and the capacity function, respectively. Let f be a flow on this network and (A, B) a cut. Show that

$$\sum_{\substack{(u,v)\in E(\Gamma)\\ u\in A\\ v\in B}} f(u,v) - \sum_{\substack{(u,v)\in E(\Gamma)\\ u\in B\\ v\in A}} f(u,v) = \sum_{\substack{v\in V(\Gamma)\\ (S,v)\in E(\Gamma)}} f(S,v) - \sum_{\substack{u\in V(\Gamma)\\ (u,S)\in E(\Gamma)}} f(u,S).$$

5. [extra 2pt] Let $\mathbf{v}_1, \dots, \mathbf{v}_5$ be the columns of

$$\begin{bmatrix} 1 & 4 & 5 & 2 & 3 \\ 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 2 & 1 \end{bmatrix}.$$

Let $\mathbf{e}_1 = (1, 0, 0)^\top$. Find a subset $S \subseteq \{1, 2, 3, 4, 5\}$ such that $\{\mathbf{v}_i\}_{i \in S}$ is a basis and

$$\sum_{i \in S} \langle \mathbf{e}_1, \mathbf{v}_i \rangle$$

is minimized. Here $\langle \mathbf{x}, \mathbf{y} \rangle$ is the standard inner product in $\mathbb{R}^3$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	