

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (一)

MATH 103A / GEAI 1215A: Linear Algebra I

第一次期中考

October 3, 2022

Midterm 1

姓名 Name : _____

學號 Student ID # : _____

Lecturer: Jephian Lin 林晉宏

Contents: cover page,
5 pages of questions,
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. Let

$$\mathbf{u} = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

(a) [1pt] Find the length of \mathbf{u} .

(b) [1pt] Let θ be the angle between the vectors \mathbf{u} and \mathbf{v} . Find $\cos \theta$.

(c) [1pt] Find a vector in $\text{span}(\{\mathbf{u}, \mathbf{v}\})$. Provide your reasons.

(d) [1pt] Find a vector **NOT** in $\text{span}(\{\mathbf{u}, \mathbf{v}\})$. Provide your reasons.

(e) [1pt] Find a nonzero vector that is orthogonal to both \mathbf{u} and \mathbf{v} .

2. Let

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & -6 & 4 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

(a) [2pt] Draw the vector \mathbf{b} and the subspace $\text{Col}(A)$ in \mathbb{R}^2 . Mark at least one vector in $\text{Col}(A)$ in your drawing. Does $A\mathbf{x} = \mathbf{b}$ have a solution for \mathbf{x} ?

(b) [1pt] Draw the subspace $\ker(A)$ and its normal vector in \mathbb{R}^3 .

(c) [2pt] Find \mathbf{h}_1 and \mathbf{h}_2 such that $\ker(A) = \text{span}(\{\mathbf{h}_1, \mathbf{h}_2\})$.

3. Let

$$A = \begin{bmatrix} 1 & 5 & -4 & -2 \\ -1 & -5 & 4 & 3 \\ 2 & 10 & -8 & -6 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -5 \\ 6 \\ -12 \end{bmatrix}.$$

(a) [2pt] Find the reduced echelon form of the augmented matrix $[A \mid \mathbf{b}]$.

(b) [3pt] Find \mathbf{p} , \mathbf{h}_1 , and \mathbf{h}_2 such that the set of general solutions of $A\mathbf{x} = \mathbf{b}$ is

$$\{\mathbf{p} + c_1\mathbf{h}_1 + c_2\mathbf{h}_2 : c_1, c_2 \in \mathbb{R}\}.$$

4. Suppose you are talking to people who have never learned linear algebra. Follow the guidelines below and try to explain the concept of a *subspace* as clear as possible.

(a) [3pt] Define what is a subspace and use a few sentences to explain the definition.

(b) [2pt] Provide an example of a subspace and an example of a set that is not a subspace. Provide your reasons.

5. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 \\ 1 & 1 & 3 & 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 3 \\ 5 \\ t \end{bmatrix}.$$

Find t such that the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	