國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（一） MATH 103A / GEAI 1215A: Linear Algebra I

第二次期中考 November 14, 2022 **Midterm 2**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. [1pt] Find a 3×4 matrix A with $\text{rank}(A) = 1$ such that every entry of A is nonzero. Provide your reasons.

2. [1pt] Find a 3×4 matrix A with $\text{null}(A) = 1$ such that every entry of A is nonzero. Provide your reasons.

3. [1pt] Find a polynomial of degree at most 2 that is in $\text{span}\{1-x, 1-x^2\}$.

4. [1pt] Find a polynomial of degree at most 2 that is NOT in $\text{span}\{1-x, 1-x^2\}$.

5. [1pt] Write $p(x) = 1+x+x^2$ as a linear combination of $\{1, 1-x, (1-x)^2\}$.

6. Let

$$A = \begin{bmatrix} 1 & -3 & -2 & 1 \\ 2 & -6 & -4 & 3 \\ -5 & 15 & 10 & -10 \end{bmatrix} \text{ and } R = \begin{bmatrix} 1 & -3 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

such that R is the reduced echelon form of A.

(a) [1pt] Find a basis of $\mathrm{Row}(A)$.

(b) [1pt] Find a basis of $\mathrm{Col}(A)$.

(c) [2pt] Find a basis of $\ker(A)$.

(d) [1pt] Find a basis of $\ker(A^\top)$.

7. Let V be a subspace with a basis $\beta = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$. Let $\alpha = \{\mathbf{a}_1, \mathbf{a}_2\}$ be a subset in V. Suppose it is known that $\mathbf{a}_1 = \mathbf{b}_1 + 2\mathbf{b}_3$ and $\mathbf{a}_2 = \mathbf{b}_1 + 3\mathbf{b}_3$.

   (a) [1pt] Write $\mathbf{b}_1$ as a linear combination of $\{\mathbf{a}_1, \mathbf{b}_2, \mathbf{b}_3\}$.

   (b) [2pt] Show that $\{\mathbf{a}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is linearly independent. (This is an important step in the proof of the basis exchange lemma, so please do not use the lemma to prove this statement.)

   (c) [2pt] Find a basis S of V such that $\alpha \subseteq S \subseteq \alpha \cup \beta$. (It is okay to use the basis exchange lemma here if needed.)

8. Suppose you are talking to people who have never learned linear algebra. Follow the guidelines below and try to explain the concept of a *basis* as clear as possible.

   (a) [3pt] Suppose V is a subspace. Define what is a basis of V and use a few sentences to explain the definition.

   (b) [2pt] Let $V = \mathbb{R}^3$. Provide an example of a basis of V and an example of a subset of V that is not a basis. Provide your reasons.

9. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Find a basis of $\mathrm{Col}(A) \cap \mathrm{Col}(B)$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	