姓名 Name　:　_____

學號 Student ID #　:　_____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.

- Using the calculator is not allowed (and not necessary) for this exam.

- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

- 可用中文或英文作答

1. [1pt] What is the elementary matrix corresponding the row operation $\rho_1 \leftrightarrow \rho_3$ applied on matrices with 3 rows? What is its determinant?

2. [1pt] What is the elementary matrix corresponding the row operation $\rho_3 : \times 3$ applied on matrices with 3 rows? What is its determinant?

3. [1pt] What is the elementary matrix corresponding the row operation $\rho_1 : +7\rho_3$ applied on matrices with 3 rows? What is its determinant?

4. [2pt] Find a 4×4 matrix A such that $\det(A) = 7$ and every entry of A is nonzero. (Explain why your answer is correct.)

5. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 16 \\ 1 & 3 & 9 & 27 & 81 \\ 1 & 5 & 25 & 125 & 625 \\ 1 & 6 & 36 & 216 & 1296 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 0 & 0 & x \\ 1 & 2 & 4 & 8 & 16 \\ 1 & 3 & 9 & 27 & 81 \\ 1 & 5 & 25 & 125 & 625 \\ 1 & 6 & 36 & 216 & 1296 \end{bmatrix}$$

(a) [1pt] Find $\det(A)$.

(b) [2pt] Find the last row of A^{-1}.

(c) [2pt] Find the x such that $\det(B) = 0$.

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6. Show that $\det(A) = \det(A^\top)$ for any square matrix A.

7. [5pt] 數學作文：請寫一篇短文來向沒修過線性代數的朋友介紹什麼是行列式值（determinant）。

請敘述行列式值的定義，並解釋定義中每一條規則的直觀意義。請以自己的方式、盡量白話的敘述、或是比喻來說明為什麼要考慮這樣的概念？請給一些能幫助他人理解的例子，並提出一些這個概念的相關性質；有必要的話可以加上一些圖來輔助說明。格式沒有限制，篇幅大約半面到一面。

(If Chinese is not your native language, you may use English or the language that you prefer.)

8. [extra 2pt] Let A be the 9×9 matrix

$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix} .
$$

Find $\det(A)$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	