國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（二） **MATH 104 / GEAI 1209: Linear Algebra II**

第一次期中考 March 21, 2022 **Midterm 1**

姓名 Name : solution

學號 Student ID # : ____________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **5 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- 可用中文或英文作答

1. [1pt] What is the elementary matrix corresponding the row operation $\rho_1 \leftrightarrow \rho_2$ applied on matrices with 3 rows? What is its determinant?

$$E = \begin{pmatrix} & 1 & \\ 1 & & \\ & & 1 \end{pmatrix} \qquad \det(E) = -1$$

2. [1pt] What is the elementary matrix corresponding the row operation $\rho_2 : \times 2$ applied on matrices with 3 rows? What is its determinant?

$$E = \begin{pmatrix} 1 & & \\ & 2 & \\ & & 1 \end{pmatrix} \qquad \det(E) = 2$$

3. [1pt] What is the elementary matrix corresponding the row operation $\rho_1 : +5\rho_2$ applied on matrices with 3 rows? What is its determinant?

$$E = \begin{pmatrix} 1 & 5 & \\ & 1 & \\ & & 1 \end{pmatrix} \qquad \det(E) = 1$$

4. [2pt] Find a 4×4 matrix A such that $\det(A) = 5$ and every entry of A is nonzero. (Explain why your answer is correct.)

$$\begin{pmatrix} 5 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \xrightarrow{\times 1} \begin{pmatrix} 5 & 1 & 1 & 1 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \xrightarrow{+1} \begin{pmatrix} 5 & 1 & 1 & 1 \\ 5 & 2 & 1 & 1 \\ 5 & 1 & 2 & 1 \\ 5 & 1 & 1 & 2 \end{pmatrix}$$

5. Let

$$A=\begin{bmatrix}1&1&1&1&1\\1&2&4&8&16\\1&3&9&27&81\\1&4&16&64&256\\1&6&36&216&1296\end{bmatrix}\quad\text{and } B=\begin{bmatrix}1&0&0&0&x\\1&2&4&8&16\\1&3&9&27&81\\1&4&16&64&256\\1&6&36&216&1296\end{bmatrix}$$

(a) [1pt] Find $\det(A)$.

Vandermonde mtx.

$$\det(A) = (2-1)(3-2)(4-3)(6-4)(3-1)(4-2)(6-3)\cdot(4-1)(6-2)(6-1)$$
$$= 1\cdot 1\cdot 1\cdot 2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 4\cdot 5\,.$$
$$= \cancel{720}\ \underline{1440}$$

(b) [2pt] Find the last row of A^{-1}.

last row of A^{-1} = last column of $A^{cof} / \det(A)$.

$$1,5\text{-cofactor} = 1\cdot\frac{(3-2)(4-3)(6-4)(4-2)(6-3)(6-2)}{\det(A)} = \frac{1}{(2-1)(3-1)(4-1)(6-1)} = \frac{1}{30}$$

$$2,5\text{-cofactor} = (-1)\cdot\frac{1}{(2-1)(3-2)(4-2)(6-2)} = -\frac{1}{8}$$

$$3,5\text{-cofactor} = 1\cdot\frac{1}{(3-1)(3-2)\cancel{(3-)}(4-3)(6-3)} = \frac{1}{6}$$

$$4,5\text{-cofactor} = (-1)\cdot\frac{1}{(4-1)(4-2)(4-3)(6-4)} = -\frac{1}{12}$$

$$5,5\text{-cofactor} = 1\cdot\frac{1}{(6-1)(6-2)(6-3)(6-4)} = \frac{1}{\cancel{60}\ 120}$$

$\Rightarrow$ last row $= \left(\frac{1}{30}, -\frac{1}{8}, \frac{1}{6}, -\frac{1}{12}, \cancel{\frac{1}{60}}\ \frac{1}{120}\right)$

(c) [2pt] Find the x such that $\det(B) = 0$.

$$\det(B) = \det\underbrace{\begin{pmatrix}2&4&8&16\\3&\cdots\\4&\cdots\\6&\cdots\end{pmatrix}}_{X} + x\cdot\det\underbrace{\begin{pmatrix}1&2&\cdots\\1&3&\cdots\\1&4&\cdots\\1&6&\cdots\end{pmatrix}}_{Y} = \det(X) + x\det(Y)$$

因 $X = \begin{pmatrix}2&&&\\&3&&\\&&4&\\&&&6\end{pmatrix} Y$，故 $\det(X) = 2\cdot 3\cdot 4\cdot 6\cdot\det(Y)$.

若 $\det(B)=0$，则 $x = -\det(X)/\det(Y) = -2\cdot 3\cdot 4\cdot 6 = \underline{-144}$.

6. Show that $\det(A) = \det(A^\top)$ for any square matrix A.

See ver A

7. [5pt] 數學作文：請寫一篇短文來向沒修過線性代數的朋友介紹什麼是行列式值（determinant）。

請敘述行列式值的定義，並解釋定義中每一條規則的直觀意義。請以自己的方式、盡量白話的敘述、或是比喻來說明為什麼要考慮這樣的概念？請給一些能幫助他人理解的例子，並提出一些這個概念的相關性質；有必要的話可以加上一些圖來輔助說明。格式沒有限制，篇幅大約半面到一面。

(If Chinese is not your native language, you may use English or the language that you prefer.)

8. [extra 2pt] Let A be the 9×9 matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Find $\det(A)$.

See ver. A

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	