國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（一） **MATH 103A / GEAI 1215A: Linear Algebra I**

第一次期中考 October 2, 2023 **Midterm 1**

姓名 Name : solution

學號 Student ID # : ____________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. [5pt] Consider the three points

$$A = (2, 2, 0, 0, 0, 0),$$
$$B = (0, 0, 2, 2, 0, 0),$$
$$C = (0, 0, 1, 1, 1, 1).$$

Draw the triangle ABC on this paper as accurate as possible. Mark the length of the three sides and calculate the three angles.

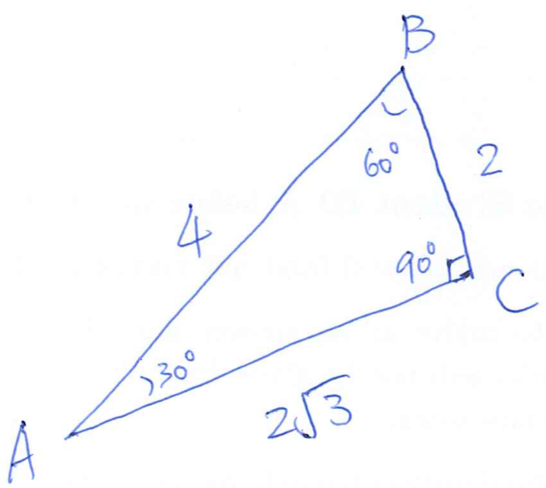
$$\overline{AB} = \|(2,2,-2,-2,0,0)\| = \sqrt{4+4+4+4} = 4.$$

$$\overline{AC} = \|(2,2,-1,-1,-1,-1)\| = \sqrt{4+4+1+1+1+1} = 2\sqrt{3}$$

$$\overline{BC} = \|(0,0,1,1,-1,-1)\| = \sqrt{1+1+1+1} = 2.$$

It is a triangle with sides $2 : 4 : 2\sqrt{3}$
$$\shortparallel$$
$1 : 2 : \sqrt{3}$,

So its a $30°$-$60°$-$90°$ triangle.



2. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 3 & 3 & 1 \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ 4 \\ 3 \end{bmatrix}, \text{ and } \mathbf{y} = \begin{bmatrix} 3 \\ -4 \\ 4 \\ -3 \end{bmatrix}.$$

(a) [1pt] Is $\mathbf{x}$ in $\ker(A)$?

No, $\vec{x} \notin \ker(A)$ since $A\vec{x} = \begin{bmatrix} 14 \\ 22 \\ 15 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

(b) [1pt] Is $\mathbf{y}$ in $\ker(A)$?

Yes, $\vec{y} \in \ker(A)$ since $A\vec{y} = \vec{0}$.

(c) [1pt] Is $\mathbf{x}$ in $\mathrm{Row}(A)$?

Yes. Let $\vec{r_1} = (1, 1, 1, 1)$
$\vec{r_2} = (1, 2, 2, 1)$
$\vec{r_3} = (1, 3, 3, 1)$.

Solve the equations $c_1\vec{r_1} + c_2\vec{r_2} + c_3\vec{r_3} = \vec{x}$

and get $\begin{cases} c_1 + c_2 + c_3 = 3 \\ c_1 + 2c_2 + 3c_3 = 4 \\ c_1 + 2c_2 + 3c_3 = 4 \\ c_1 + c_2 + c_3 = 3 \end{cases}$

and $\vec{x} = 2\vec{r_1} + 1\vec{r_2} + 0\vec{r_3}$,

so $\vec{x} \in \mathrm{Row}(A)$.

(d) [1pt] Is $\mathbf{y}$ in $\mathrm{Row}(A)$?

No. The equations
$c_1\vec{r_1} + c_2\vec{r_2} + c_3\vec{r_3} = \vec{y}$
have no solution.

(e) [1pt] Describe the relation between $\ker(A)$ and $\mathrm{Row}(A)$.

They are orthogonal to each other.

3. [5pt] Find all solutions of the following system of linear equations.

$$\begin{cases} x - 2y + 5u = 1 \\ 2x - 4y + z - 3w + 9u = 3 \\ -8x + 16y - 3z + 9w - 37u = -11 \end{cases}$$

Consider the augmented mtx

$$\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 5 & 1 \\ 2 & -4 & 1 & -3 & 9 & 3 \\ -8 & 16 & -3 & 9 & -37 & -11 \end{array}\right] \xrightarrow{\text{row operation}} \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 5 & 1 \\ 0 & 0 & 1 & -3 & -1 & 1 \\ 0 & 0 & -3 & 9 & 3 & -3 \end{array}\right]$$

$$\begin{array}{c} \quad x \quad\ y \quad\ z \quad w \quad u \\ \longrightarrow \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 5 & 1 \\ 0 & 0 & 1 & -3 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]. \end{array}$$

leading : x, z.

free : y, w, u.

① Solve particular solution from $A\vec{x} = \vec{b}$ by setting $y = w = u = 0$.

$$\Rightarrow \quad \vec{p} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

② Solve the homogeneous solution from $A\vec{x} = \vec{0}$

- by setting $y=1, w=0, u=0$. $\Rightarrow \vec{h}_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

- by setting $y=0, w=1, u=0 \Rightarrow \vec{h}_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}$.

- by setting $y=0, w=0, u=1 \Rightarrow \vec{h}_3 = \begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$.

Solutions = $\underline{\vec{p} + \operatorname{span}(\{\vec{h}_1, \vec{h}_2, \vec{h}_3\})}$

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the notion of span(S).

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. [extra 2pt] Let $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}$ be vectors in $\mathbb{R}^n$ such that $\mathbf{w} = \mathbf{x} + \mathbf{y} + \mathbf{z}$. Show that $\mathbf{p} = 100\mathbf{x} + 200\mathbf{y} + 300\mathbf{z}$ is in $\operatorname{span}(\{\mathbf{x}, \mathbf{y}, \mathbf{w}\})$.

$$\vec{p} = 100\vec{x} + 200\vec{y} + 300(\vec{w} - \vec{x} - \vec{y})$$

$$= -200\vec{x} - 100\vec{y} + 300\vec{w}.$$

So $\vec{p} \in \operatorname{span}(\{\vec{x}, \vec{y}, \vec{w}\})$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	