國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（一） **MATH 103A / GEAI 1215A: Linear Algebra I**

第二次期中考 November 13, 2023 **Midterm 2**

姓名 Name : solution

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

– Enter your **Name** and **Student ID #** before you start.

– Using the calculator is not allowed (and not necessary) for this exam.

– Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

– Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

– Please answer the problems in English.

1. [1pt] Let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : \cancel{x}\ y = 1 \right\}$. Is V a subspace in $\mathbb{R}^2$? Provide your reason.

No. We have $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in V$

but $\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \notin V$.

2. [1pt] Let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : (x-y)^2 = 0 \right\}$. Is V a subspace in $\mathbb{R}^2$? Provide your reason.

Yes. $V = \{(0,0)\}$ Satisfies the three conditions of a subspace.

3. Let

$$A = \begin{bmatrix} 3 & 3 & 6 & 6 \\ 1 & 1 & 2 & 2 \\ 4 & 4 & 5 & 5 \end{bmatrix}.$$

(a) [1pt] Find a basis of Row(A).

$\{(3,3,6,6), (4,4,5,5)\}$,

or

$\{(1,1,0,0), (0,0,1,1)\}$.

(b) [1pt] Find a basis of Col(A).

$\left\{ \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix} \right\}$.

(c) [1pt] Find rank(A) and null(A).

rank $(A) = 2$.

null $(A) = 2$

4. [3pt] Let

$$S = \{(x-2)(x-4), (x-2)(x-6), (x-4)(x-6)\}.$$

Show that S is linearly independent, or provide a certificate of S not being independent.

We show that S is linearly independent.

Suppose $c_1(x-2)(x-4) + c_2(x-2)(x-6) + c_3(x-4)(x-6) = 0$

Plug in $x=2$. Then $c_3 = 0$.

$x=4$. Then $c_2 = 0$.

$x=6$. Then $c_1 = 0$.

So S is linearly independent.

5. [2pt] Let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$ be vectors in $\mathbb{R}^3$. Suppose we know

$$\text{span}(\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}) = \text{span}(\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}).$$

What is the relation between $\mathbf{u}_4$ and the set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$? Provide an example to demonstrate this behavior.

See ver. A.

6. [5pt] Let

$$A = \begin{bmatrix} 1 & -1 & -5 & 3 & 7 \\ 2 & -2 & -10 & 7 & 15 \\ -4 & 4 & 20 & -13 & -29 \end{bmatrix}.$$

Find a basis of $\ker(A)$.

See ver. A.

7. [5pt] Mathematical essay: Write a few paragraphs to introduce the notion of a basis.

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

8. [extra 2pt] Let V be the space of all functions defined on $(0, \infty)$. Determine if $S = \{\ln(x), \ln(x^2)\}$ is linearly independent or not.

See ver. A.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	