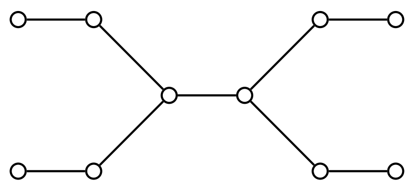
2023F Math585 Midterm 2

5 questions, $20(+5)$ **total points**
Note: Use other papers to answer the problems. Remember to write down your **name** and your **student ID #**.

1. [5pt] Let C_n be the cycle on n vertices and A_n its adjacency matrix. For $n \geqslant 0$, find the $1,1$-entry of $(A_{n+1})^n$.

2. [5pt] Let G be the graphs below and A its adjacency matrix. Find $\operatorname{rank}(A)$, $\det(A)$, and the inertia of A.



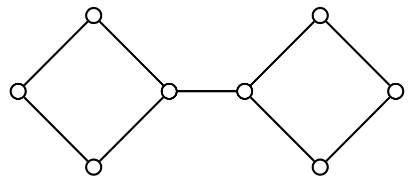
3. [5pt] Let

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

Find $\operatorname{spec}(A)$.

Two more problems on the back.

4. [5pt] Let G be the graphs below and A its adjacency matrix.



Find the characteristic polynomial $\det(A - xI)$ of A.

5. [extra 5pt] Let

$$A = \begin{bmatrix} O_{m\times m} & B \\ C & O_{n\times n} \end{bmatrix},$$

where O is the zero matrix of the designated order. For $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^m$. Show that $\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$ is an eigenvector of A with respect to λ if and only if $\begin{bmatrix} \mathbf{x} \\ -\mathbf{y} \end{bmatrix}$ is an eigenvector of A with respect to $-\lambda$.