

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (二)

MATH 104A / GEAI 1209A: Linear Algebra II

期末考

May 29, 2023

Final Exam

姓名 Name : _____

學號 Student ID # : _____

Lecturer: Jephian Lin 林晉宏

Contents: cover page,
6 pages of questions,
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 7 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

5. [5pt] Let

$$A = \begin{bmatrix} -18 & 1 & 2 & 3 \\ -45 & 3 & 5 & 7 \\ -63 & 4 & 7 & 10 \\ -81 & 5 & 9 & 13 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

It is known that \mathbf{v} is an eigenvector of A . Find the spectrum of A .

6. [5pt] Let

$$A = \begin{bmatrix} 4 & 1 & 1 & 1 & 1 \\ 1 & 4 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 & 2 \end{bmatrix}.$$

Find the spectrum of A . You may also give some partial information of the spectrum of A to get some partial credits, e.g., $\lambda_1 \geq 10$.

7. Let A be an $m \times n$ matrix and $A = U\Sigma V^\top$ its singular value decomposition. Suppose you are talking to people who have never learned linear algebra. Follow the guidelines below and try to explain the concept of the *singular value decomposition* as clear as possible.

(a) [2pt] Explain what U , Σ and V are, including their sizes and properties.

(b) [2pt] Let \mathbf{u}_i 's and \mathbf{v}_i 's be the columns of U and V , respectively. Describe their relations.

(c) [1pt] Give some reasons about why the singular value decomposition is important.

8. [extra 5pt] Let A be the 9×9 matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}.$$

Find the inertia of A .

9. [extra 2pt] Let

$$f(x, y) = x^2 + 4xy + 4y^2.$$

Find the maximum value of $f(x, y)$ subject to $x^2 + y^2 = 1$.

[END]

Page	Points	Score
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4	5	
5	5	
6	2	
Total	20 (+7)	