

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (二)

MATH 104A / GEAI 1209A: Linear Algebra II

第一次期中考

March 20, 2023

Midterm 1

姓名 Name : _____

學號 Student ID # : _____

Lecturer: Jephian Lin 林晉宏

Contents: cover page,
5 pages of questions,
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. [1pt] Write down the 3×3 elementary matrix for the row operation $\rho_3 : +5\rho_2$ and find its determinant.

2. [1pt] Write down the 3×3 elementary matrix for the row operation $\rho_2 \leftrightarrow \rho_3$ and find its determinant.

3. [1pt] Write down the 3×3 elementary matrix for the row operation $\rho_1 : \times 3$ and find its determinant.

4. [2pt] Find the adjugate of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

5. [2pt] Find the determinant of

$$A = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 \end{bmatrix}.$$

6. [3pt] Find the determinant of

$$L = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix}.$$

7. Let

$$A_x = \begin{bmatrix} -x & 1 & 1 & 1 & 1 \\ 1 & -x & 0 & 0 & 0 \\ 1 & 0 & -x & 0 & 0 \\ 1 & 0 & 0 & -x & 0 \\ 1 & 0 & 0 & 0 & -x \end{bmatrix}.$$

(a) [2pt] Find $\det(A_x)$.

(b) [3pt] Find all x such that $\det(A_x) = 0$. For each of such x , find a nonzero vector \mathbf{v} in $\ker(A_x)$.

8. [5pt] Let

$$S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}.$$

Let E be a 2×2 elementary matrix. Discuss how E changes the shape of S into $ES = \{E\mathbf{v} : \mathbf{v} \in S\}$ and calculate its area. Make sure you consider each of the three types of elementary matrices and give some concrete examples.

9. [extra 2pt] Let

$$A = \begin{bmatrix} - & \mathbf{x} & - \\ - & \mathbf{y} & - \\ - & \mathbf{z} & - \end{bmatrix} \text{ and } B = \begin{bmatrix} - & \mathbf{x} + \mathbf{y} & - \\ - & \mathbf{y} + 2\mathbf{z} & - \\ - & \mathbf{z} + 3\mathbf{x} & - \end{bmatrix}$$

be 3×3 matrices. Suppose $\det(A) = 1$. Find $\det(B)$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	