

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (二)

MATH 104A / GEAI 1209A: Linear Algebra II

第一次期中考

March 20, 2023

Midterm 1

姓名 Name : \_\_\_\_\_

學號 Student ID # : \_\_\_\_\_

Lecturer: Jephian Lin 林晉宏

Contents: cover page,  
**5 pages** of questions,  
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 2 extra points

**Do not open this packet until instructed to do so.**

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. [1pt] Write down the  $3 \times 3$  elementary matrix for the row operation  $\rho_1 : \times 2$  and find its determinant.
  
  
  
  
  
  
  
  
  
  
2. [1pt] Write down the  $3 \times 3$  elementary matrix for the row operation  $\rho_3 : +4\rho_2$  and find its determinant.
  
  
  
  
  
  
  
  
  
  
3. [1pt] Write down the  $3 \times 3$  elementary matrix for the row operation  $\rho_1 \leftrightarrow \rho_3$  and find its determinant.
  
  
  
  
  
  
  
  
  
  
4. [2pt] Find the adjugate of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

5. [2pt] Find the determinant of

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

6. [3pt] Find the determinant of

$$L = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}.$$

7. Let

$$A_x = \begin{bmatrix} -x & 1 & 1 & 1 & 1 \\ 1 & -x & 0 & 0 & 0 \\ 1 & 0 & -x & 0 & 0 \\ 1 & 0 & 0 & -x & 0 \\ 1 & 0 & 0 & 0 & -x \end{bmatrix}.$$

(a) [2pt] Find  $\det(A_x)$ .

(b) [3pt] Find all  $x$  such that  $\det(A_x) = 0$ . For each of such  $x$ , find a nonzero vector  $\mathbf{v}$  in  $\ker(A_x)$ .

8. [5pt] Let

$$S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \leq 1 \right\}.$$

Let  $E$  be a  $2 \times 2$  elementary matrix. Discuss how  $E$  changes the shape of  $S$  into  $ES = \{E\mathbf{v} : \mathbf{v} \in S\}$  and calculate its area. Make sure you consider each of the three types of elementary matrices and give some concrete examples.

9. [extra 2pt] Let

$$A = \begin{bmatrix} - & \mathbf{x} & - \\ - & \mathbf{y} & - \\ - & \mathbf{z} & - \end{bmatrix} \text{ and } B = \begin{bmatrix} - & \mathbf{x} + \mathbf{y} & - \\ - & \mathbf{y} + 2\mathbf{z} & - \\ - & \mathbf{z} + 3\mathbf{x} & - \end{bmatrix}$$

be  $3 \times 3$  matrices. Suppose  $\det(A) = 1$ . Find  $\det(B)$ .

**[END]**

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	