國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（二） **MATH 104A / GEAI 1209A: Linear Algebra II**

第二次期中考 April 24, 2023 **Midterm 2**

姓名 Name : ____________________

學號 Student ID # : ____________________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	6 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 7 extra points

Do not open this packet until instructed to do so.

Instructions:

– Enter your **Name** and **Student ID #** before you start.

– Using the calculator is not allowed (and not necessary) for this exam.

– Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

– Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

– Please answer the problems in English.

1. Let

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

   (a) [1pt] Find an eigenvector of A and write down its corresponding eigenvalue.

   (b) [1pt] Find a nonzero vector that is not an eigenvector of A.

   (c) [1pt] Find A^{100}.

   (d) [1pt] Find the characteristic polynomial of A.

   (e) [1pt] Find the spectrum of A.

2. Suppose we know that

$$\begin{bmatrix} 0 & 1 \\ 6 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 3/5 & 1/5 \\ 2/5 & -1/5 \end{bmatrix}.$$

(a) [3pt] Find the formula of a_n in the recurrence relation

$$\begin{cases} a_{n+2} &= -a_{n+1} + 6a_n, \\ a_0 &= 1, \ a_1 = 0. \end{cases}$$

(b) [2pt] Let x_1 and x_2 be functions in t. Solve the system of differential equations. (Remember to add constants at appropriate places.)

$$\begin{cases} \dot{x}_1 &= \qquad x_2, \\ \dot{x}_2 &= 6x_1 \ -x_2. \end{cases}$$

3. [5pt] Let

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

Find the characteristic polynomial and the spectrum of A.

4. [5pt] Suppose we know that

$$\begin{bmatrix} 73 & 36 \\ 36 & 52 \end{bmatrix} = \begin{bmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{bmatrix} \begin{bmatrix} 25 & 0 \\ 0 & 100 \end{bmatrix} \begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix}.$$

Describe the ellipse defined by

$$73x^2 + 72xy + 52y^2 = 1,$$

including the directions and the length of its axes. Provide reasons to your answers.

5. [extra 2pt] Let A be the 10×10 matrix

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

Let

$$\det(A - xI) = a_0 x^{10} + a_1 x^9 + a_2 x^8 + \cdots + a_{10}$$

be its characteristic polynomial. Find a_2.

6. [extra 5pt] Let

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

(a) Find an invertible matrix Q and a diagonal matrix D such that $A = QDQ^{-1}$.

(b) Find an orthogonal matrix Q and a diagonal matrix D such that $A = QDQ^{\top}$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
6	5	
Total	20 (+7)	