姓名 Name : _____

學號 Student ID # : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	6 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** $+$ 7 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.

- Using the calculator is not allowed (and not necessary) for this exam.

- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

- Please answer the problems in English.

1. Let
$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 4 & 4 \end{bmatrix} \text{ and } R = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

It is known that R is an echelon form of A.

(a) [1pt] Find at least two elements in $\ker(A)$.

(b) [1pt] Find at least two elements **not** in $\ker(A)$.

(c) [1pt] Find a basis of $\mathrm{Row}(A)$.

(d) [1pt] Find a basis of $\ker(A)$.

(e) [1pt] Find a basis of $\mathrm{Col}(A)$.

2. Let $\mathcal{P}_2 = \{a_0 + a_1 x + a_2 x^2 : a_0, a_1, a_2 \in \mathbb{R}\}$ be the vector space of polynomial of degree at most 2. Let

$$B_1 = \langle 1, x, x^2 \rangle, \ B_2 = \langle 1, (x+1), (x+1)^2 \rangle, \ B_3 = \langle f_1(x), f_2(x), f_3(x) \rangle,$$

where

$$f_1(x) = \frac{(x-3)(x-5)}{(1-3)(1-5)}, \ f_2(x) = \frac{(x-1)(x-5)}{(3-1)(3-5)}, \ f_3(x) = \frac{(x-1)(x-3)}{(5-1)(5-3)}.$$

It is known that each of B_1, B_2, B_3 is a basis of $\mathcal{P}_2$. Let

$$p(x) = 1 + 2x + 3x^2 \text{ and } \mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(a) [1pt] Find $\operatorname{Rep}_{B_1}(p(x))$.

(b) [1pt] Find $\operatorname{Rep}_{B_2}(p(x))$.

(c) [1pt] Find $\operatorname{Rep}_{B_3}(p(x))$.

(d) [1pt] Find a polynomial $q(x)$ such that $\operatorname{Rep}_{B_2}(q(x)) = \mathbf{u}$.

(e) [1pt] Find a polynomial $q(x)$ such that $\operatorname{Rep}_{B_3}(q(x)) = \mathbf{u}$.

3. Let $\mathcal{M}_{2\times2}$ be the vector space of all 2×2 real matrices. Let

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$$

be a subspace of $\mathcal{M}_{2\times2}$.

(a) [1pt] Find a basis of $\mathcal{M}_{2\times2}$.

(b) [1pt] Find at least two elements in V.

(c) [3pt] Find a basis of V. Hint: Write down the equations for a, b, c, d.

4. [5pt] Mathematical essay: Write a few paragraphs to introduce *dimension*.

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. Let $B = \langle \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4 \rangle$ be a basis of a vector space V. Suppose $\mathbf{p} = \mathbf{u}_1 + \mathbf{u}_4$. Let

$$B_1 = \langle \mathbf{u}_1, \mathbf{u}_2, \mathbf{p}, \mathbf{u}_4 \rangle, \text{ and}$$
$$B_2 = \langle \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{p} \rangle.$$

(a) [extra 1pt] Explain why B_1 is not a basis of V.

(b) [extra 2pt] Show that B_2 is independent.

(c) [extra 2pt] Show that $\text{span}(B_2) = V$.

6. [extra 2pt] Let

$$V = \{(x+1)(a + bx + cx^2) : a, b, c \in \mathbb{R}\}.$$

Find a basis of V.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	2	
Total	20 (+7)	