姓名 Name   : _____

學號 Student ID #   : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.

- Using the calculator is not allowed (and not necessary) for this exam.

- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

- Please answer the problems in English.

1. For the following problems, use a, b, c as the variables.

   (a) [1pt] Give an example of an equation that is **not** linear.

   (b) [1pt] Give an example of a system of linear equations that has no solution.

   (c) [1pt] Give an example of a system of linear equations that has a unique solution.

   (d) [1pt] Give an example of a system of linear equations that has infinitely many solutions.

   (e) [1pt] Give an example of a system of linear equations that is **not** in the echelon form.

2. Consider the following system of linear equations.

$$\begin{cases} x + 3y + 3z + 4w = 13 \\ 4x + 12y + 13z + 17w = 56 \\ -2x - 6y - 6z - 7w = -26 \end{cases}$$

(a) [1pt] Find the echelon form.

(b) [1pt] List all the leading variables.

(c) [1pt] List all the free variables.

(d) [1pt] Find a solution to the system.

(e) [1pt] Find a solution to the associated **homogeneous** system.

3. [5pt] Consider the following system of linear equations.

$$\begin{cases} x + 5y - 3z + 2u - 15v = -3 \\ 3x + 15y - 8z + 4u - 37v = -2 \\ 4x + 20y - 11z + 7u - 55v = -9 \end{cases}$$

Solve the system and describe the solution set in the form of

$$\{\mathbf{p} + c_1\boldsymbol{\beta}_1 + c_2\boldsymbol{\beta}_2 : c_1, c_2 \in \mathbb{R}\}.$$

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the *solution set* of a system of linear equations.

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. [extra 2pt] Consider the following system of linear equations.

$$\begin{cases} 2x - y - z = a \\ -x + 2y - z = b \\ -x - y + 2z = c \end{cases}$$

For what condition(s) on a, b, c, the system has at least one solution?

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	