國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數(一)

MATH 103A / GEAI 1215A: Linear Algebra I

第一次期中考

October 16, 2024

Midterm 1

姓名 Name : _____

學號 Student ID # : ___ solution

Lecturer: Jephian Lin 林晉宏

Contents: cover page,

Duration: 110 minutes

5 pages of questions,

score page at the end

To be answered: on the test paper

Total points: 20 points + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your Name and Student ID # before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by underlining it or circling it. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

- 1. For the following problems, use a, b, c as the variables.
 - (a) [1pt] Give an example of an equation that is **not** linear.

(b) [1pt] Give an example of a system of linear equations that has no solution.

$$\begin{cases} a+b+c=1\\ a+b+c=2 \end{cases}$$

(c) [1pt] Give an example of a system of linear equations that has a unique solution.

$$\begin{cases} \alpha & = 1 \\ b & = 2 \end{cases}$$

(d) [1pt] Give an example of a system of linear equations that has infinitely many solutions.

$$\begin{cases} a+b+c=1 \\ \vdots \end{cases}$$

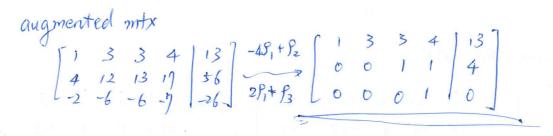
(e) [1pt] Give an example of a system of linear equations that is **not** in the echelon form.

$$\begin{cases} a+b+c=1\\ a+b+c=1 \end{cases}$$

2. Consider the following system of linear equations.

$$\begin{cases} x + 3y + 3z + 4w = 13 \\ 4x + 12y + 13z + 17w = 56 \\ -2x - 6y - 6z - 7w = -26 \end{cases}$$

(a) [1pt] Find the echelon form.



(b) [1pt] List all the leading variables.

(c) [1pt] List all the free variables.

(d) [1pt] Find a solution to the system.

$$\begin{array}{c} e.g. \\ \begin{cases} y\\ z \\ w \end{cases} = \begin{bmatrix} 1\\ 0\\ 4\\ 0 \end{bmatrix}$$

(e) [1pt] Find a solution to the associated homogeneous system.

Solve
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e.g. $\begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$

3. [5pt] Consider the following system of linear equations.

$$\begin{cases} x + 5y - 3z + 2u - 15v = -3\\ 3x + 15y - 8z + 4u - 37v = -2\\ 4x + 20y - 11z + 7u - 55v = -9 \end{cases}$$

Solve the system and describe the solution set in the form of

$$\{\mathbf{p} + c_1 \boldsymbol{\beta}_1 + c_2 \boldsymbol{\beta}_2 : c_1, c_2 \in \mathbb{R}\}.$$

Solve the homogeneous system for \$, \$

Set
$$y_0=1$$
, $v=0$, $\overrightarrow{\beta}_1=\begin{bmatrix} -5\\ 0\\ 0\\ 0\end{bmatrix}$

Set
$$y=0, v=1$$
, $\beta_{2} = \begin{bmatrix} 3 \\ 0 \\ -2 \\ 3 \end{bmatrix}$

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the *solution set* of a system of linear equations.

Your score will be based on the following criteria.

- The definition is clear.
- Some sentences are added to explain the definition.
- Examples or pictures are included to help understanding.
- The sentences are complete.

5. [extra 2pt] Consider the following system of linear equations.

$$\begin{cases} 2x - y - z = a \\ -x + 2y - z = b \\ -x - y + 2z = c \end{cases}$$

For what condition(s) on a, b, c, the system has at least one solution?

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	Page	Points	Score
	1	5	
	2	5	
	3	5	
	4	5	
B	5	2	
	Total	20 (+2)	