國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（一） **MATH 103A / GEAI 1215A: Linear Algebra I**

第一次期中考 October 16, 2024 **Midterm 1**

姓名 Name : _____

學號 Student ID # : solution

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **5 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. For the following problems, use a, b, c as the variables.

   (a) [1pt] Give an example of an equation that is **not** linear.

   $a^2+b^2+c^2=1$

   (b) [1pt] Give an example of a system of linear equations that has no solution.

   $\begin{cases} a+b+c=1 \\ a+b+c=2 \end{cases}$

   (c) [1pt] Give an example of a system of linear equations that has a unique solution.

   $\begin{cases} a=1 \\ b=2 \\ c=3 \end{cases}$

   (d) [1pt] Give an example of a system of linear equations that has infinitely many solutions.

   $\{a+b+c=1$

   (e) [1pt] Give an example of a system of linear equations that is **not** in the echelon form.

   $\begin{cases} a+b+c=1 \\ a+b+c=1 \end{cases}$

2. Consider the following system of linear equations.

$$\begin{cases} x + 3y + 3z + 4w = 13 \\ 4x + 12y + 13z + 17w = 56 \\ -2x - 6y - 6z - 7w = -26 \end{cases}$$

(a) [1pt] Find the echelon form.

augmented mtx

$$\left[\begin{array}{cccc|c} 1 & 3 & 3 & 4 & 13 \\ 4 & 12 & 13 & 17 & 56 \\ -2 & -6 & -6 & -7 & -26 \end{array}\right] \xrightarrow[2P_1+P_3]{-4P_1+P_2} \left[\begin{array}{cccc|c} 1 & 3 & 3 & 4 & 13 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

(b) [1pt] List all the leading variables.

x, z, w

(c) [1pt] List all the free variables.

y

(d) [1pt] Find a solution to the system.

e.g. $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \\ 0 \end{bmatrix}$

(e) [1pt] Find a ~~non-zero~~ solution to the associated **homogeneous** system.

Solve $\left[\begin{array}{cccc|c} 1 & 3 & 3 & 4 & 0 \\ & & 1 & 1 & 0 \\ & & & 1 & 0 \end{array}\right]$

e.g. $\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

3. [5pt] Consider the following system of linear equations.

$$\begin{cases} x + 5y - 3z + 2u - 15v = -3 \\ 3x + 15y - 8z + 4u - 37v = -2 \\ 4x + 20y - 11z + 7u - 55v = -9 \end{cases}$$

Solve the system and describe the solution set in the form of

$$\{\mathbf{p} + c_1\boldsymbol{\beta}_1 + c_2\boldsymbol{\beta}_2 : c_1, c_2 \in \mathbb{R}\}.$$

augmented mtx

$$\left[\begin{array}{ccccc|c} 1 & 5 & -3 & 2 & -15 & -3 \\ 3 & 15 & -8 & 4 & -37 & -2 \\ 4 & 20 & -11 & 7 & -55 & -9 \end{array}\right] \xrightarrow{\substack{-3R_1+R_2 \\ -4R_1+R_3}} \left[\begin{array}{ccccc|c} 1 & 5 & -3 & 2 & -15 & -3 \\ 0 & 0 & 1 & -2 & 8 & 7 \\ 0 & 0 & 1 & -1 & 5 & 3 \end{array}\right]$$

$$\xrightarrow{-R_2+R_3} \left[\begin{array}{ccccc|c} 1 & 5 & -3 & 2 & -15 & -3 \\ 0 & 0 & 1 & -2 & 8 & 7 \\ 0 & 0 & 0 & 1 & -3 & -4 \end{array}\right] \quad \cdots (*)$$

Solve (*) for $\vec{p}$: free vars are y, v.

Set $y, v = 0$, $\vec{p} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ -4 \\ 0 \end{bmatrix}$

Solve the homogeneous system for $\vec{\beta}_1, \vec{\beta}_2$.

Set $y = 1$, $v = 0$, $\vec{\beta}_1 = \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Set $y = 0$, $v = 1$, $\vec{\beta}_2 = \begin{bmatrix} 3 \\ 0 \\ -2 \\ 3 \\ 1 \end{bmatrix}$

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the *solution set* of a system of linear equations.

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. [extra 2pt] Consider the following system of linear equations.

$$\begin{cases} 2x - y - z = a \\ -x + 2y - z = b \\ -x - y + 2z = c \end{cases}$$

For what condition(s) on a, b, c, the system has at least one solution?

augmented mtx

$$\left[\begin{array}{ccc|c} 2 & -1 & -1 & a \\ -1 & 2 & -1 & b \\ -1 & -1 & 2 & c \end{array}\right] \xrightarrow{\substack{f_1+f_3 \\ f_2+f_3}} \left[\begin{array}{ccc|c} 2 & -1 & -1 & a \\ -1 & 2 & -1 & b \\ 0 & 0 & 0 & a+b+c \end{array}\right]$$

$$\xrightarrow{2f_2+f_1} \left[\begin{array}{ccc|c} 0 & 3 & -3 & a+2b \\ -1 & 2 & -1 & b \\ 0 & 0 & 0 & a+b+c \end{array}\right] \xrightarrow{f_1 \leftrightarrow f_2} \left[\begin{array}{ccc|c} -1 & 2 & -1 & b \\ 0 & 3 & -3 & a+2b \\ 0 & 0 & 0 & a+b+c \end{array}\right]$$

As long as $a+b+c=0$, the system has a solution.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	