

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (一)

MATH 103A / GEAI 1215A: Linear Algebra I

第一次期中考

October 16, 2024

Midterm 1

姓名 Name : _____

學號 Student ID # : _____

Lecturer: Jephian Lin 林晉宏

Contents: cover page,
5 pages of questions,
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. For the following problems, use p, q, r as the variables.

(a) [1pt] Give an example of an equation that is **not** linear.

(b) [1pt] Give an example of a system of linear equations that has no solution.

(c) [1pt] Give an example of a system of linear equations that has a unique solution.

(d) [1pt] Give an example of a system of linear equations that has infinitely many solutions.

(e) [1pt] Give an example of a system of linear equations that is **not** in the echelon form.

2. Consider the following system of linear equations.

$$\begin{cases} x + 2y + 5z - 3w = -4 \\ 2x + 5y + 14z - 5w = -10 \\ 2x + 2y + 2z - 7w = -4 \end{cases}$$

(a) [1pt] Find the echelon form.

(b) [1pt] List all the leading variables.

(c) [1pt] List all the free variables.

(d) [1pt] Find a solution to the system.

(e) [1pt] Find a solution to the associated **homogeneous** system.

3. [5pt] Consider the following system of linear equations.

$$\begin{cases} x + 5y - 3z + 2u - 15v = -3 \\ 3x + 15y - 8z + 4u - 37v = -2 \\ 4x + 20y - 11z + 7u - 55v = -9 \end{cases}$$

Solve the system and describe the solution set in the form of

$$\{\mathbf{p} + c_1\boldsymbol{\beta}_1 + c_2\boldsymbol{\beta}_2 : c_1, c_2 \in \mathbb{R}\}.$$

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the *solution set* of a system of linear equations.

Your score will be based on the following criteria.

- The definition is clear.
- Some sentences are added to explain the definition.
- Examples or pictures are included to help understanding.
- The sentences are complete.

5. [extra 2pt] Consider the following system of linear equations.

$$\begin{cases} 2x - y - z = a \\ -x + 2y - z = b \\ -x - y + 2z = c \end{cases}$$

For what condition(s) on a, b, c , the system has at least one solution?

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	