國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（一） **MATH 103A / GEAI 1215A: Linear Algebra I**

第一次期中考 October 16, 2024 **Midterm 1**

姓名 Name : _____

學號 Student ID # : solution

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **5 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. For the following problems, use p, q, r as the variables.

   (a) [1pt] Give an example of an equation that is **not** linear.

   $pqr = 3$.

   (b) [1pt] Give an example of a system of linear equations that has no solution.

   $$\begin{cases} 0p+0q+0r=3 \end{cases}$$

   (c) [1pt] Give an example of a system of linear equations that has a unique solution.

   $$\begin{cases} p+q+r=1 \\ q+r=2 \\ r=3 \end{cases}$$

   (d) [1pt] Give an example of a system of linear equations that has infinitely many solutions.

   $$\begin{cases} 0p+0q+0r=0 \end{cases}$$

   (e) [1pt] Give an example of a system of linear equations that is **not** in the echelon form.

   $$\begin{cases} p+q+r=0 \\ 2p+2q+2r=0 \end{cases}$$

2. Consider the following system of linear equations.

$$\begin{cases} x + 2y + 5z - 3w = -4 \\ 2x + 5y + 14z - 5w = -10 \\ 2x + 2y + 2z - 7w = -4 \end{cases}$$

(a) [1pt] Find the echelon form.

augmented mtx.

$$\left[\begin{array}{cccc|c} 1 & 2 & 5 & -3 & -4 \\ 2 & 5 & 14 & -5 & -10 \\ 2 & 2 & 2 & -7 & -4 \end{array}\right] \xrightarrow[-2P_1+P_3]{-2P_1+P_2} \left[\begin{array}{cccc|c} 1 & 2 & 5 & -3 & -4 \\ 0 & 1 & 4 & 1 & -2 \\ 0 & -2 & -8 & -1 & 4 \end{array}\right]$$

$$\xrightarrow{2P_2+P_3} \left[\begin{array}{cccc|c} 1 & 2 & 5 & -3 & -4 \\ 0 & 1 & 4 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

(b) [1pt] List all the leading variables.

x, y, w

(c) [1pt] List all the free variables.

z

(d) [1pt] Find a solution to the system.

e.g. $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 0 \end{bmatrix}$

(e) [1pt] Find a solution to the associated **homogeneous** system.

e.g. $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 1 \\ 0 \end{bmatrix}$

3. [5pt] Consider the following system of linear equations.

$$\begin{cases} x - 3y + 3z + 8u - 2v = -17 \\ 3x - 9y + 10z + 28u - 9v = -55 \\ -2x + 6y - 6z - 16u + 5v = 34 \end{cases}$$

Solve the system and describe the solution set in the form of

$$\{\mathbf{p} + c_1\boldsymbol{\beta}_1 + c_2\boldsymbol{\beta}_2 : c_1, c_2 \in \mathbb{R}\}.$$

See ver A.

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the *solution set* of a system of linear equations.

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. [extra 2pt] Consider the following system of linear equations.

$$\begin{cases} 2x - y - z = a \\ -x + 2y - z = b \\ -x - y + 2z = c \end{cases}$$

For what condition(s) on a, b, c, the system has at least one solution?

See ver. A.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	