

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (一)

MATH 103A / GEAI 1215A: Linear Algebra I

第二次期中考

November 20, 2024

Midterm 2

姓名 Name : _____

學號 Student ID # : _____

Lecturer: Jephian Lin 林晉宏

Contents: cover page,
5 pages of questions,
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. Let

$$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy < 0 \right\}.$$

(a) [1pt] List any four elements in V .

(b) [1pt] Is V closed under addition? Provide your reasons.

(c) [1pt] Is V closed under scalar multiplication? Provide your reasons.

(d) [1pt] Does V contain a zero vector? If yes, what is the zero vector in V ?

(e) [1pt] Find the additive inverse of $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

2. Let $S = \{\mathbf{u}_1, \mathbf{u}_2\}$ with

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \text{ and } \mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}.$$

(a) [1pt] Find an element in $[S]$.

(b) [1pt] Find an element not in $[S]$.

(c) [1pt] Prove, by definition, that $[S]$ is closed under addition.

(d) [1pt] Prove, by definition, that $[S]$ is closed scalar multiplication.

(e) [1pt] Is S an independent set? Provide your reasons.

3. Let $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5, \mathbf{u}_6\}$ be the columns of

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 1 & 2 \\ 1 & 2 & 4 & 1 & 1 & 2 \\ 1 & 3 & 6 & 2 & 1 & 3 \\ 1 & 4 & 8 & 1 & 2 & 3 \end{bmatrix}$$

(a) [2pt] Find two different nontrivial linear relations in S .

(b) [3pt] Find a subset $T \subset S$ such that T is linearly independent and $[T] = [S]$. Provide your reasons.

4. [5pt] Mathematical essay: Write a few paragraphs to introduce *linear independence*.

Your score will be based on the following criteria.

- The definition is clear.
- Some sentences are added to explain the definition.
- Examples or pictures are included to help understanding.
- The sentences are complete.

5. [extra 2pt] Let $S = \{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ be a set of vectors. Suppose S is linearly independent. Show that $T = \{\mathbf{x}, \mathbf{x} + \mathbf{y}, \mathbf{x} + \mathbf{y} + \mathbf{z}\}$ is also linearly independent.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	