

姓名 Name : \_\_\_\_\_ 學號 Student ID # : \_\_\_\_\_

Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 & -12 \\ 2 & 5 & 3 & -28 \\ 12 & 29 & 19 & -164 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 2$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 4 & -4 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_3, x_4$ .

By setting  $x_4 = 1$  and all other free variables as 0, one may solve for

$$\beta_2 = \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of  $\beta_2$ ) mod 10 = 9.

NullBasis 1



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code

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姓名 Name : \_\_\_\_\_ 學號 Student ID # : \_\_\_\_\_

Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & 5 & -4 & -19 \\ -1 & -4 & 4 & 16 \\ -5 & -21 & 20 & 83 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 1$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -4 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_3, x_4$ .

By setting  $x_3 = 1$  and all other free variables as 0, one may solve for

$$\beta_1 = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of  $\beta_1$ ) mod 10 = 5.

NullBasis 2



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code

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Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & -8 & 3 \\ 2 & 9 & -19 & 6 \\ 13 & 56 & -116 & 39 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 1$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_3, x_4$ .

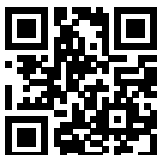
By setting  $x_3 = 1$  and all other free variables as 0, one may solve for

$$\beta_1 = \begin{bmatrix} -4 \\ 3 \\ 1 \\ 0 \end{bmatrix}.$$

as the answer.

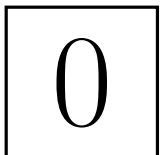
Check code = (sum of all entries of  $\beta_1$ ) mod 10 = 0.

NullBasis 3



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code



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Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & -4 & -10 \\ -4 & 13 & 17 & 42 \\ 16 & -53 & -69 & -170 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 2$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -1 & -4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_3, x_4$ .

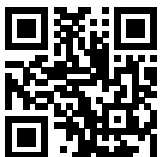
By setting  $x_4 = 1$  and all other free variables as 0, one may solve for

$$\beta_2 = \begin{bmatrix} 4 \\ -2 \\ 0 \\ 1 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of  $\beta_2$ ) mod 10 = 3.

NullBasis 4



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code

3

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Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & 5 \\ 3 & -3 & 0 & 16 \\ 1 & -1 & 0 & 7 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 1$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_2, x_3$ .

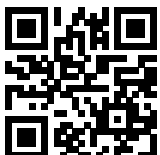
By setting  $x_2 = 1$  and all other free variables as 0, one may solve for

$$\beta_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of  $\beta_1$ ) mod 10 = 2.

NullBasis 5



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code

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Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & -4 & 2 & -1 \\ -5 & 20 & -10 & 6 \\ 10 & -40 & 20 & -13 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 2$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -4 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_2, x_3$ .

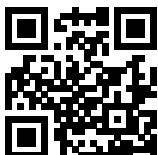
By setting  $x_3 = 1$  and all other free variables as 0, one may solve for

$$\beta_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of  $\beta_2$ ) mod 10 = 9.

NullBasis 6



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code

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Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & -5 & -24 \\ -4 & 12 & 21 & 101 \\ 19 & -57 & -99 & -476 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 2$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_2, x_4$ .

By setting  $x_4 = 1$  and all other free variables as 0, one may solve for

$$\beta_2 = \begin{bmatrix} -1 \\ 0 \\ -5 \\ 1 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of  $\beta_2$ ) mod 10 = 5.

NullBasis 7



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code

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Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & -5 & 15 & 11 \\ -4 & 21 & -63 & -46 \\ 5 & -27 & 81 & 59 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 1$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_3, x_4$ .

By setting  $x_3 = 1$  and all other free variables as 0, one may solve for

$$\beta_1 = \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of  $\beta_1$ ) mod 10 = 4.

NullBasis 8



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code

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Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & -3 & 3 \\ -3 & -12 & 10 & -10 \\ 20 & 80 & -65 & 65 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 2$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_2, x_4$ .

By setting  $x_4 = 1$  and all other free variables as 0, one may solve for

$$\beta_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of  $\beta_2$ ) mod 10 = 2.

NullBasis 9



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code

2

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Quiz 1

MATH 103: Linear Algebra I

Consider the equation  $\mathbf{Ax} = \mathbf{0}$ , where

$$\mathbf{A} = \begin{bmatrix} 1 & -5 & 1 & 2 \\ 4 & -20 & 4 & 9 \\ -13 & 65 & -13 & -28 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form  $\mathbf{R}$  of  $\mathbf{A}$  to get the free variables. Let  $k = 2$ . Find a solution  $\mathbf{x} = \beta_k$  by setting the  $k$ -th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of  $\beta_k$ ) mod 10

**Solution.**

Apply Gaussian elimination to  $\mathbf{A}$  to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are  $x_2, x_3$ .

By setting  $x_3 = 1$  and all other free variables as 0, one may solve for

$$\beta_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of  $\beta_2$ ) mod 10 = 0.

NullBasis 10



Indicating your answer by **underlining it** or **circling it**.  
Compute the **check code** and fill it into the **box on the right**.

check code

