姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{Ax} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 & -12 \\ 2 & 5 & 3 & -28 \\ 12 & 29 & 19 & -164 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 4 & -4 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_4 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} 4 \\ 4 \\ 0 \\ 1 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 9.

NullBasis 1

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

9

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 5 & -4 & -19 \\ -1 & -4 & 4 & 16 \\ -5 & -21 & 20 & 83 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -4 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boldsymbol{\beta}_1 = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 5.

NullBasis 2

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

5

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & -8 & 3 \\ 2 & 9 & -19 & 6 \\ 13 & 56 & -116 & 39 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_1 = \begin{bmatrix} -4 \\ 3 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 0.

NullBasis 3

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

0

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & -4 & -10 \\ -4 & 13 & 17 & 42 \\ 16 & -53 & -69 & -170 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & -1 & -4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_4 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} 4 \\ -2 \\ 0 \\ 1 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 3.

NullBasis 4

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

3

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & 5 \\ 3 & -3 & 0 & 16 \\ 1 & -1 & 0 & 7 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_3.

By setting $x_2 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 2.

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

2

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{Ax} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -4 & 2 & -1 \\ -5 & 20 & -10 & 6 \\ 10 & -40 & 20 & -13 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -4 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_3.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 9.

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

9

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{Ax} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & -5 & -24 \\ -4 & 12 & 21 & 101 \\ 19 & -57 & -99 & -476 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -3 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_4.

By setting $x_4 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} -1 \\ 0 \\ -5 \\ 1 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 5.

NullBasis 7

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

5

姓名 Name : ____________________ 學號 Student ID # : ____________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{Ax} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -5 & 15 & 11 \\ -4 & 21 & -63 & -46 \\ 5 & -27 & 81 & 59 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 1$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_3, x_4.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_1 = \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_1$) mod 10 = 4.

NullBasis 8

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

4

姓名 Name : ______________________ 學號 Student ID # : ______________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & -3 & 3 \\ -3 & -12 & 10 & -10 \\ 20 & 80 & -65 & 65 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_4.

By setting $x_4 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 2.

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

2

姓名 Name : ________________ 學號 Student ID # : ________________

Quiz 1 MATH 103: Linear Algebra I

Consider the equation $\mathbf{A}\mathbf{x} = \mathbf{0}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & -5 & 1 & 2 \\ 4 & -20 & 4 & 9 \\ -13 & 65 & -13 & -28 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \text{ and } \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Compute the reduced echelon form $\mathbf{R}$ of $\mathbf{A}$ to get the free variables. Let $k = 2$. Find a solution $\mathbf{x} = \boldsymbol{\beta}_k$ by setting the k-th free variable as 1 while the other free variables as 0.

Check code = (sum of all entries of $\boldsymbol{\beta}_k$) mod 10

Solution.

Apply Gaussian elimination to $\mathbf{A}$ to get its reduced echelon form

$$\mathbf{R} = \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The free variables are x_2, x_3.

By setting $x_3 = 1$ and all other free variables as 0, one may solve for

$$\boxed{\boldsymbol{\beta}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}.$$

as the answer.

Check code = (sum of all entries of $\boldsymbol{\beta}_2$) mod 10 = 0.

NullBasis 10

Indicating your answer by **underlining it** or **circling it**.
Compute the **check code** and fill it into the **box on the right**.

check code

0