2024F Math589 Midterm 1

5 questions, $20(+5)$ **total points**
Note: Use other papers to answer the problems. Remember to write down your **name** and your **student ID #**.
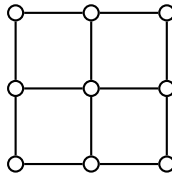
1. [5pt] Let
$$V_1 = V_2 = \{1, 2, 3, 4, 5\},$$
$$E_1 = \{\{1, 3\}, \{2, 3\}, \{3, 4\}, \{4, 5\}\}, \text{ and}$$
$$E_2 = \{\{1, 3\}, \{3, 5\}, \{4, 5\}, \{2, 5\}\}.$$

   Draw the two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ with the vertices labeled. Then provide a bijection between V_1 and V_2 to show that G_1 and G_2 are isomorphic.

2. [5pt] Let G be the graph below. Find a spanning subgraph H_1 of G that is not an induced subgraph. Also, find an induced subgraph H_2 of G that is not a spanning subgraph.

3. [5pt] Find a graph whose degree sequence is $0, 1, 2, 3, 4, 5, 6, 7, 8$ or show that such graph does not exist.

4. [5pt] Let $m, n \geqslant 1$ be integers. Let $G_{m,n} = (V, E)$ with

$$V = \{(i, j) : 1 \leqslant i \leqslant m, \text{ and } 1 \leqslant j \leqslant n\} \text{ and}$$
$$E = \{\{(i_1, j_1), (i_2, j_2)\} : |i_1 - i_2| + |j_1 - j_2| = 1\}.$$

   Determine for which m, n the graph $G_{m,n}$ contains a cycle of length mn as a spanning subgraph.

5. [5pt] Find a graph G whose vertex connectivity is $\kappa(G) = 2$ and whose edge connectivity is $\lambda(G) = 5$.