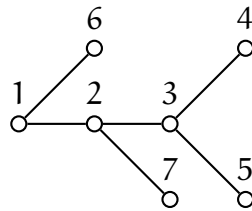# 2024F Math589 Midterm 2

5 questions, $20(+5)$ **total points**

Note: Use other papers to answer the problems. Remember to write down your **name** and your **student ID #**.

1. [5pt] Let T be the tree below with its root at $r = 1$. Find the upset $\lfloor 2 \rfloor$, the downset $\lceil 2 \rceil$, and all minimal elements in $V(T) \setminus \lceil 2 \rceil$.

2. [5pt] Recall that the d-dimensional cube Q_d is a graph whose vertex set is composed of all 0-1 sequences with d digits such that two vertices are adjacent if and only if the two sequences are different at exactly one position. For each $d \geqslant 0$, determine if Q_d is a bipartite graph. If yes, find the corresponding partition; if not, find an odd cycle.

3. [5pt] Let X be the graph below. Draw a graph H in TX that has more than 10 vertices. Also, show that H contains an IX.
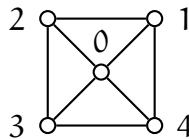
4. [5pt] Let Q_4 be the 4-dimensional cube described in Problem 2. Draw Q_4 and determine if it is Eulerian. If yes, find an Euler tour; if not, find a vertex of odd degree.

One more problem on the back.

5. [5pt] Let G be the graph below. Find a spanning normal tree of G.