國立中山大學   NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（二）   **MATH 104A / GEAI 1209A: Linear Algebra II**

第一次期中考   March 25, 2024   **Midterm 1**

姓名 Name : _____

學號 Student ID # : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

– Enter your **Name** and **Student ID #** before you start.

– Using the calculator is not allowed (and not necessary) for this exam.

– Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

– Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

– Please answer the problems in English.

1. Let $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ be vectors in $\mathbb{R}^3$. Let

$$A = \begin{bmatrix} - & \mathbf{x} & - \\ - & \mathbf{y} & - \\ - & \mathbf{z} & - \end{bmatrix}, \quad B = \begin{bmatrix} - & \mathbf{y} & - \\ - & \mathbf{x} & - \\ - & \mathbf{z} & - \end{bmatrix}, \quad C = \begin{bmatrix} - & 2\mathbf{x} & - \\ - & 3\mathbf{y} & - \\ - & 4\mathbf{z} & - \end{bmatrix},$$

$$D = \begin{bmatrix} - & \mathbf{x} & - \\ - & 3\mathbf{x} + \mathbf{y} & - \\ - & 4\mathbf{x} + \mathbf{z} & - \end{bmatrix}, \quad E = \begin{bmatrix} - & \mathbf{x} + 2\mathbf{y} & - \\ - & \mathbf{y} + 2\mathbf{z} & - \\ - & \mathbf{z} + 2\mathbf{x} & - \end{bmatrix}, \quad F = \begin{bmatrix} - & \mathbf{x} + \mathbf{z} & - \\ - & \mathbf{y} & - \\ - & \mathbf{x} + \mathbf{z} & - \end{bmatrix}.$$

Let $\det(A) = \Delta$.

(a) [1pt] Find $\det(B)$. Provide your reasons.

(b) [1pt] Find $\det(C)$. Provide your reasons.

(c) [1pt] Find $\det(D)$. Provide your reasons.

(d) [1pt] Find $\det(E)$. Provide your reasons.

(e) [1pt] Find $\det(F)$. Provide your reasons.

2. Let

$$A = \begin{bmatrix} a & b & c & d \\ 1 & 2 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 2 & 2 & 1 & 1 \end{bmatrix}.$$

(a) [4pt] Find $\det(A)$ in terms of variables a, b, c, and d.

(b) [1pt] Find some **nonzero** values of a, b, c and d such that $\det(A)$ is zero.

3. [5pt] Let
$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & 12 \\ 12 & -9 & 28 \end{bmatrix}.$$

Write A as a product of elementary matrices.

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & 12 \\ 12 & -9 & 28 \end{bmatrix}.$$

4. [5pt] Mathematical essay: Write a few paragraphs to introduce *permutation expansion*.

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

be a 10×10 matrix. Find $\det(A)$.

[**END**]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	